

Assessment Materials for Multiplicative Thinking

Assessment Task Booklet Option 1

GROWING Mathematically

Based on the *Scaffolding Numeracy in the Middle Years*
and *Reframing Mathematical Futures II* Research projects



ASSESSING MULTIPLICATIVE THINKING

ASSESSMENT TASK BOOKLET FOR MULTIPLICATIVE THINKING OPTION 1

NAME
YEAR LEVEL:

This booklet contains 5 tasks, an Extended Task and 4 Supplementary or Short Tasks:

X1 - Tables and Chairs

S1 - Butterfly House

S2 - Tiles, Tiles, Tiles

S3 - Adventure Camp

S4 - Stained Glass Windows

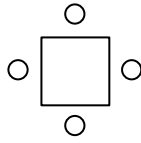
INSTRUCTIONS:

1. Please do as much of each task as you can.
2. All working must be shown in this booklet. If you need more space, please use the back of the page, but make sure we know where to find your answer.
3. When you are asked to **show all your working and explain your answer in as much detail as possible** or to **explain your reasoning using as much mathematics as you can** or **show all your working so we can understand your thinking** do your best to write down what you did and why, in the space provided.
4. If you have any questions please ask your teacher.

TABLES AND CHAIRS ...



Dean's community is planning a street party. They have lots of small square tables. Each table seats 4 people like this:



The community decides to put the tables in an end-to-end line along the street to make one big table.

- Make or draw a line with 2 tables. How many people will be able to sit at it?
- Make or draw a line of 4 tables. How many people will be able to sit at it?
- Make or draw a line of tables that would seat 8 people. How many tables are needed?
- Make or draw a line of tables that would seat 12 people. How many tables are needed?
- Make or draw a line of tables that would seat 20 people. How many tables are needed?

f. Fill in the shaded boxes to show your results so far.

Number of Tables	1	2		4					
Number of people	4		8		12				20

g. Can you find another way to describe your results so far? Show this in the space below.

h. The community can borrow 99 tables. How many people could they seat using 99 tables placed end-to-end? **Show your working and explain your answer in as much detail as possible.**

i. The community can borrow rectangular tables that seat 6 people. Draw one of these tables showing the people sitting around it.

j. Draw a line of 5 of these rectangular tables placed end-to-end. How many people will be able to sit at it?

- k. Explain what happens to the number of people as more rectangular tables are placed end-to-end. Describe or show your findings in at least two ways.
- l. How many people could be seated if 46 of these rectangular tables were placed end-to-end? **Show your working and explain your answer in as much detail as possible.**
- m. How many of these rectangular tables would you need to place end-to-end to seat 342 people? **Show your working and explain your answer in as much detail as possible.**

BUTTERFLY HOUSE...



Some children visited the Butterfly House at the Zoo.

They learnt that a butterfly is made up of 4 wings, one body and two feelers.



While they were there, they made models and answered some questions.

For each question, explain your working and your answer, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies?

_____ wings

_____ bodies

_____ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers?

- c. How many wings, bodies and feelers will be needed to make 98 model butterflies. **Show all your working and explain your answer in as much detail as possible.**

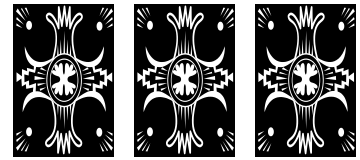
_____ wings

_____ bodies

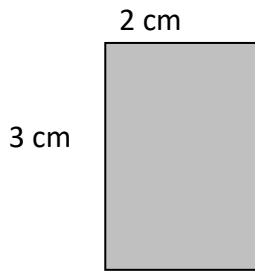
_____ feelers

- d. How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers? **Show all your working and explain your answer in as much detail as possible.**

TILES, TILES, TILES...

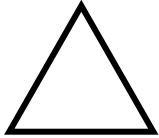
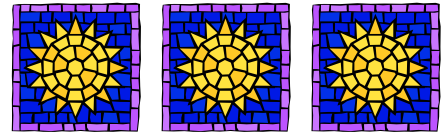


Floor and wall tiles come in different sizes. The basic tile is shown below.

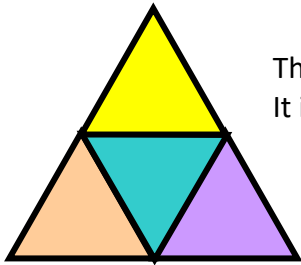


- How many basic tiles would be needed for an area of 6 cm by 4 cm?
- How many basic tiles would be needed for an area of 27cm by 18 cm?
- If the length and width of the basic tile were increased by 2 cm, how many of the larger tiles would be needed to cover 1 square metre (100 cm by 100 cm)?
Show all your working so we can understand your thinking.

STAINED GLASS WINDOWS...



Stained glass windows can be made using small triangles.



This stained glass window is made from four small triangles joined together. It is 2 triangles wide at the base and 2 triangles high.

a How many small triangles will you need if your window is to be 4 triangles wide and 4 triangles high?

b. Part of the stained glass window shown below, is hidden by a sign. How many small triangles were needed to make this window?



c. How would you advise a friend on how to work out the number of small triangles that would be needed for a window 26 triangles wide?

ASSESSING MULTIPLICATIVE THINKING

SCORING RUBRIC OPTION 1

TABLES AND CHAIRS...		
TASK:	RESPONSE:	SCORE
a.	Incorrect	0
	Correct (6)	1
b.	Incorrect	0
	Correct (10)	1
c.	Incorrect	0
	Correct (3)	1
d.	Incorrect	0
	Correct (5)	1
e.	Incorrect	0
	Correct (9)	1
f.	No response or incorrect	0
	Correct (all gaps filled correctly)	1
g.	No response or incorrect	0
	Results shown as a list or set of ordered pairs, or described additively (e.g., "It goes up by 2 each time")	1
	Results shown as a graph or expressed as a rule in words and/or symbols (e.g., "The number of people is double the number of tables plus 2 more")	2
h.	No response or incorrect	0
	Incorrect but working and/or explanation indicates attempted use of general rule, or correct (200 people) with no working and/or explanation or evidence of an additive approach (e.g., all tables drawn, or all ordered pairs listed)	1
	Correct (200 people), working and/or explanation indicates general rule recognized in some way (e.g., "I doubled it and added 2")	2
i.	No response or incorrect (e.g., does not show 6 people and/or table not rectangular)	0
	Correct, that is, rectangular table and either 2 people on longer sides and 1 on each end (arrangement A), or 3 on each of the longer sides (arrangement B)	1

j.	No response or incorrect for the arrangement indicated in the previous question	0
	Diagram correct and number of people appropriate for the arrangement shown (i.e., 22 for arrangement A or 30 for arrangement B)	1
k.	No response or incorrect, e.g., explanation irrelevant and/or findings not shown in any meaningful way	0
	Recording (i.e., explanation and/or mode(s) of representing findings) indicative of a systematic but inefficient approach consistent with arrangement shown earlier (e.g., list or diagram, or “It goes up by 4s” (or “6s” if arrangement B))	1
	Recording describes relationship between people and tables in words (e.g., “You times the number of tables by 4 and add 2” (arrangement A)), or findings shown in a way that indicates an understanding of the general rule (e.g., the inclusion of a larger number of rectangular tables and the corresponding number of people in a table or list)	2
	Recording describes relationship between people and tables expressed in symbols or equations consistent with arrangement shown earlier (e.g., $p = 6n$ for arrangement B)	3
l.	No response or incorrect	0
	Incorrect but working and/or explanation indicates use of general rule consistent with table arrangement, or correct (186 for arrangement A or 276 for arrangement B) with no working and/or explanation or evidence of an additive approach (e.g., makes all, counts all or an inefficient repeated addition strategy)	1
	Correct (186 for arrangement A or 276 for arrangement B), working and/or explanation indicates efficient strategy based on general rule	2
m.	No response or incorrect	0
	Incorrect but working and/or explanation indicates use of general rule (e.g., divide by 6 for arrangement B), but no appreciation that an extra table is needed for remaining number of people	1
	Correct (85 tables for arrangement A or 57 tables for arrangement B) with no working and/or explanation or evidence of an additive approach (e.g., all tables drawn, or all ordered pairs listed)	2
	Correct (85 tables for arrangement A or 57 tables for arrangement B), working and/or explanation indicates rule recognized in some way (e.g., “I took of 2 and divided it by 4” for arrangement A)	3

BUTTERFLY HOUSE ...		
TASK:	RESPONSE:	SCORE
a.	No response or incorrect	0
	Correct (28 wings, 7 bodies, 14 feelers)	1
b.	No response or incorrect	0
	Correct (4 butterflies)	1
c.	No response or incorrect	0
	Partially correct with some indication of multiplicative thinking (e.g., multiplication algorithm attempted), or correct with little/no working, or correct but evidence of additive thinking (e.g., $98+98+98+98$)	1
	All correct (392 wings, 98 bodies, 196 feelers) with evidence of multiplicative thinking, e.g., algorithm applied correctly or efficient computation strategies such as doubling or renaming (e.g., $400-8$ for 4×98)	2
d.	No response or incorrect	0
	Correct (6 butterflies) but working and/or explanation indicative of additive thinking (e.g., make-all, count all strategy), or incorrect with some indication that the task has been understood in terms of multiplication or division	1
	Correct (6 butterflies) with clear explanation in terms of other body parts (e.g., "Can't be 7 because not enough feelers")	2

TILES, TILES, TILES ...		
TASK:	RESPONSE:	SCORE
a.	No response or incorrect with no working and/or explanation	0
	Incorrect (2 tiles), reasoning based on perceived relationship between dimensions (e.g., “2 goes into 4, 2 times and 3 goes into 6, 2 times”) or incorrect drawing, or correct but little/no working or reasoning	1
	Correct (4 tiles), with appropriate diagram and/or explanation	2
b.	No response or incorrect with little/no working and/or explanation	0
	Incorrect (9 or 18 tiles), reasoning based on factors as above, or correct (81 tiles) but little/no working/explanation	1
	Correct (81 tiles), with appropriate diagram and/or evidence of additive strategy (e.g., count all or skip count)	2
	Correct (81 tiles), with appropriate diagram and/or explanation indicating multiplicative reasoning (e.g., factors used appropriately)	3
c.	No response or incorrect with little/no working and/or explanation	0
	Some attempt (e.g., dimension of larger tile (4cm by 5cm) indicated) and /or incomplete solution attempt (e.g., attempt to draw all)	1
	Incorrect, calculation based on incorrect dimension of larger tile (e.g., 4cm by 6cm), but supported by correct reasoning of the area required; or correct (500 tiles), with little/no explanation	2
	Correct (500 tiles), supported by appropriate diagram and/or explanation based on appropriate diagram or computation strategies	3

ADVENTURE CAMP ...		
TASK:	RESPONSE:	SCORE
a.	No response or incorrect or irrelevant statement	0
	One or two relatively simple observations based on numbers alone (e.g., "Archery was the most popular activity for both Year 5 and Year 7 students", "More Year 7 students liked the rock wall than Year 5 students")	1
	At least one observation which recognises the difference in total numbers (e.g., "Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say")	2
b.	No response	0
	Incorrect (No), argument based on numbers alone (e.g., "There were 21 Year 7s and only 18 Year 5s")	1
	Correct (Yes), but little/no working or explanation to support conclusion	2
	Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals (e.g., "18 out of 75 is more than 21 out of 100"), but no formal use of fractions or percent or further argument to justify conclusion	3
	Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion (e.g., "For Year 7 it is 21%. For Year 5s, it is 24% because $18/75 = 6/25 = 24/100 = 24\%$ ")	4

STAINED GLASS WINDOWS ...		
TASK:	RESPONSE:	SCORE
a.	No response or incorrect with no working and/or explanation	0
	Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation	1
	Correct (16 triangles), with evidence of additive reasoning based on drawing and counting	2
	Correct (16 triangles), with evidence of multiplicative reasoning based on 4 by 4	3
b.	No response or incorrect with little/no working and/or explanation	0
	Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation	1
	Correct (81 triangles), with evidence of additive reasoning based on drawing and counting, or inappropriate use of area formula (e.g., $L \times W$)	2
	Correct (81 triangles), with evidence of multiplicative reasoning based on pattern (e.g., 9 by 9)	3
c.	No response or incorrect	0
	Advice based on additive thinking (e.g., "1 less each time you go up")	1
	Correct, advice based on rule (e.g., 26×26)	2

ASSESSING MULTIPLICATIVE THINKING

STUDENT SCORE SHEET OPTION 1

Student Name:	Year Level:
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Extended Task		Score	Comments
Table and Chairs	a		
	b		
	c		
	d		
	e		
	f		
	g		
	h		
	i		
	j		
	k		
	l		
	m		
Supplementary Tasks		Score	Comments
Butterfly House	a		
	b		
	c		
	d		
Tiles, Tiles, Tiles	a		
	b		
	c		
Adventure Camp	a		
	b		
Stained Glass Windows	a		
	b		
	c		

Total Raw Score		
LAF Level		

ASSESSING MULTIPLICATIVE THINKING

LAF Raw Score Translator Option 1

The following table is provided to enable teachers to locate students on the revised **Learning and Assessment Framework for Multiplicative Thinking 2021 (LAF)** based on their performance on Assessment Option 1 (blue font indicates revisions to the original LAF).

To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 25 items altogether - 13 for Tables and Chairs, 4 for Butterfly House, 3 for Tiles, Tiles, Tiles, 2 for Adventure Camp, and 3 for Stained Glass Windows). It is inappropriate to use the raw scores for any other purpose.

Total Score	LAF Zone	Level Description
41-47	8	Can use appropriate representations, language, and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems. <i>Can express more complex multiplicative relationships in words or symbols in simplest form and work with two variables simultaneously and equivalent expressions. Recognises and uses scale appropriately and can use a generalised solution strategy in a new context. Beginning to recognise the relationships between perimeter, area, and volume.</i>
35-40	7	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent, and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically. <i>Able to describe multiplicative relationships as rules in words or symbols but may not express this in simplest form. Can reason algebraically and use symbols to describe what is needed to maintain equivalence in an additive relational context. Can use relationships to calculate simpler volumes and explain thinking in procedural terms.</i>
29-34	6	Can work with the Cartesian Product (for each) idea to systematically list or determine the number of options. Can solve a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Able to rename and compare fractions in the halving family and use partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts. <i>Able to describe and justify rules involving multiplicative relationships Beginning to generalise patterns and formalise rules involving multiplication but may miss more complex patterns involving a constant or ratio or scales requiring estimation or use of diagonals.</i>
		Systematically solves simple proportion and array problems suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions. Able to solve simple, 2-step problems using a

26-28	5	recognised rule/relationship but finds this difficult for larger numbers. Able to order numbers involving tens, ones, tenths, and hundredths in supportive context. Able to determine all options in Cartesian product situations involving relatively small numbers but tends to do this additively. Beginning to work with decimal numbers and percent but unable to apply efficiently to solve problems. Some evidence that multiplicative thinking being used to support partitioning. Beginning to approach a broader range of multiplicative situations more systematically for instance, able to recognise and apply simple ratios to solve problems involving proportion or scale.
20-25	4	Solves more familiar multiplication and division problems involving two-digit numbers. Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations. Tends not to explain their thinking or indicate working. Able to partition given number or quantity into equal parts and describe part formally. Beginning to work with simple proportion (e.g., uses % to describe a sample ; can make a start, represent problem, but unable to complete successfully or justify their thinking). Beginning to recognise and use generalisations to solve problems but unable to explain or justify thinking. Use simple scales in straightforward situations and recognises the importance of scale in more complex contexts.
16-19	3	Demonstrates intuitive sense of proportion. Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving. May list all options in a simple Cartesian product but cannot explain or justify solutions. Uses abbreviated methods for counting groups (e.g., doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions). Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems. Can maintain equivalence across the equals sign and extend patterns but may not be able to explain or explanation relies on additive thinking. Beginning to recognise the importance of scale.
11-15	2	Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting, counts large collections efficiently, systematically keeps track of count, for instance, may order groups in arrays or as a list, but needs to 'see' all groups. Can share collections into equal groups. Recognises small numbers as composite units (e.g., can count equal groups, skip count by twos, threes, and fives). Can extend an additive pattern. Recognises multiplication is relevant but tends not to be able to follow this through to solution. Can list some of the options in simple Cartesian Product and chance situations. Some evidence of MT as equal groups/shares seen as entities that can be counted systematically. Beginning to recognise statistical variation and has some understanding of chance.
1-10	1	Can solve simple multiplication and division problems involving relatively small whole numbers, but tends to rely on drawing, models, and count-all strategies. May use skip counting (repeated addition) for groups less than 5. Can make simple observations from data given in a task and extend a simple pattern number pattern. Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.