

Name:

Year Level: Date:



ALGEBRA FORM A

Assessment Booklet

Reframing Mathematical Futures II

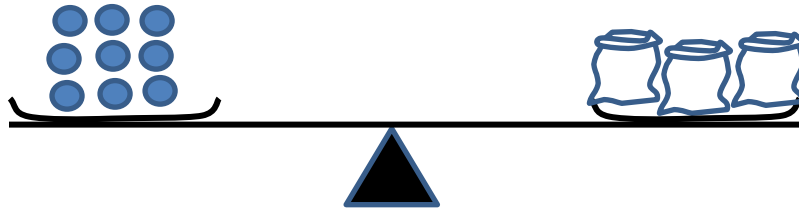
*An Australian Mathematics & Science Partnership Project
(2015-2018)*

ALGEBRA FORM A

1. Balancing Scales

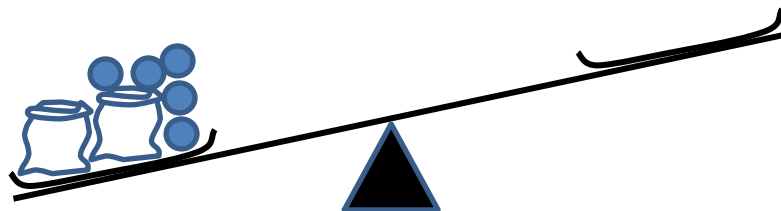
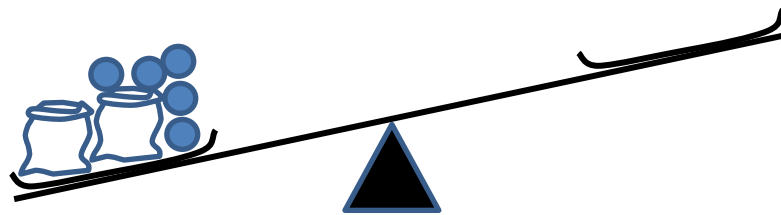
The bags on these scales all weigh the same.

The balls on these scales all weigh the same.



a [AEQB1]

Using the information above, draw two different ways to make the scales balance (you may use as many bags and/or balls as you wish).



b [AEQB2]

Write an equation for each scale you drew.

2a [ARLT1]

Put a number in the box to make this a true number sentence. Explain your reasoning.

$$347 + \boxed{} = 671 + 357$$

b [ARELS1]

What numbers would go in these boxes to make a true number sentence (the numbers may be different). Explain your reasoning.

$$\boxed{} + 521 = 527 + \boxed{}$$

c [ARELS2]

Find a different pair of numbers that would make the number sentence in part **b** true.

d [ARELS3]

Describe how you could find all possible pairs of numbers that would make this a true number sentence.

e [ARELS7]

What can you say about the relationship between c and d in this equation?

$$c \times 2 = d \times 14$$

3. Lemonade Recipe

- 3 cups of sugar,
- 4 cups of water
- juice of 12 medium sized lemons.

a [ALEM1]

How much sugar would be needed if 12 cups of water were to be used?

b [ALEM2]

Explain how much sugar and how much water would be needed for each lemon used.

c [ALEM3]

Write a rule for the amount of sugar needed for n lemons, where n represents the number of lemons. Explain your reasoning.

4. Consecutive Numbers

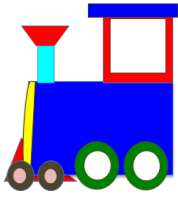
a [ACONS1]

Claudia says that “the sum of three consecutive numbers such as 15, 16, 17 is always three times the middle number”. Is she correct?

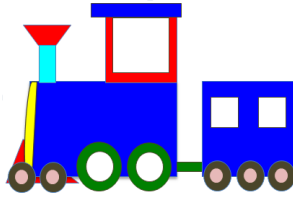
b [ACONS2]

Write the reasons for your answer.

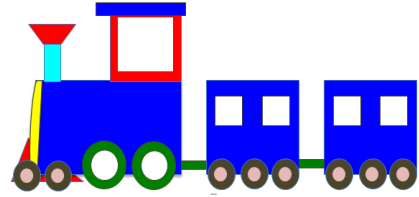
5. Trains



toy train size 1



toy train size 2



toy train size 3

The engine of the train has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each train:

Train size	1	2	3	4	5	6
Number of wheels	8	14				

a [ATRNS1]

Fill in the table to show the number of wheels for the trains size 3, 4, 5 and 6.

b [ATRNS2]

The largest train set in the toy shop is size 15.

How many wheels does the size 15 have? _____

Explain your reasoning using as much mathematics as you can.

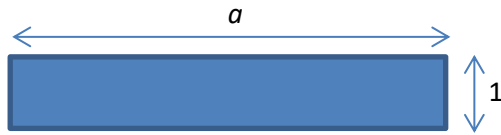
c [ATRNS3]

Ben says his train has exactly 60 wheels. Can Ben be correct? _____

Explain your reasoning using as much mathematics as you can.

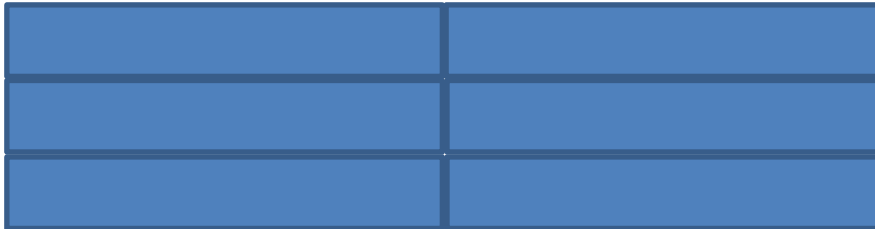
6. Tiles

This tile has length of a units and width of 1 unit.

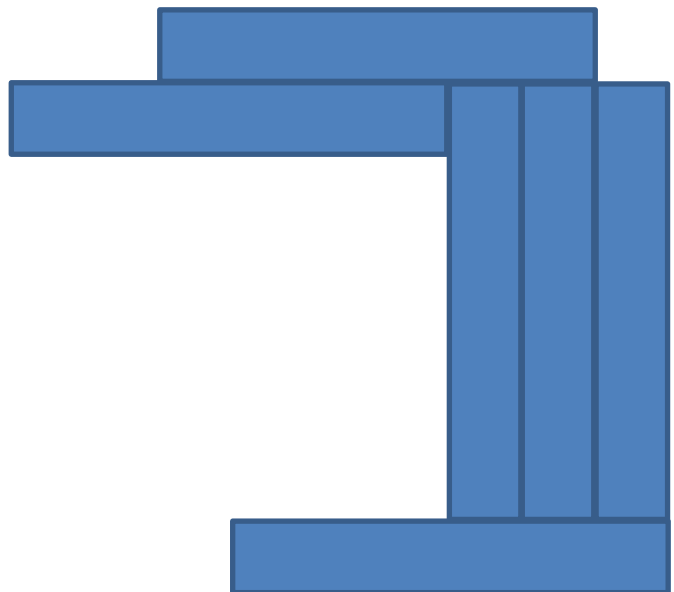


a [ATILP1]
What is the perimeter of the tile (without using a ruler)?

b [ATILP2]
The shape below is made of the same tiles. Without using a ruler, what is the perimeter of the tile?



c [ATILP3]
The shape on the right is made of the same tiles.
Again, without using a ruler, what is the perimeter of this shape?
Explain your reasoning.



7. Hot Air Balloon Trip

Jenny was given a ride in a hot air balloon for her birthday.

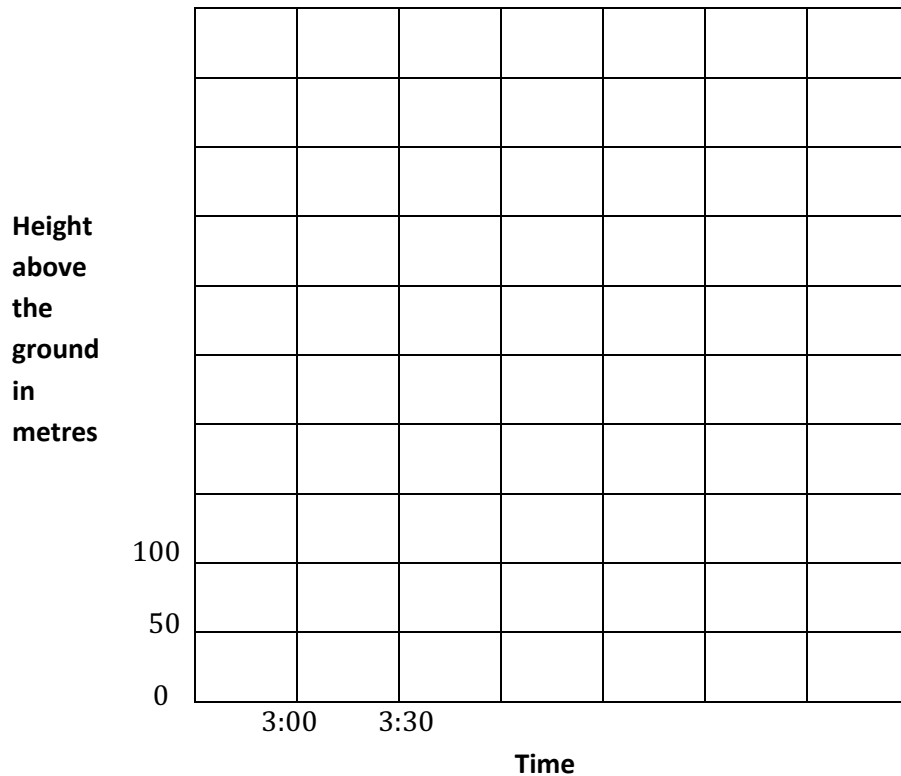
The table below shows some information about the trip.



Time	2:30 pm	3:00 pm	3:30 pm	4:00 pm	4:30 pm	5:00 pm	5:30 pm	6:00 pm
Height above ground in metres	0	150	250	350	500	250	100	0

a [AHAB1]

Complete the scale on the axis and draw the graph to show the information on the table.



b [AHAB2]

About how long did the balloon stay at or above 250 metres? _____

c [AHAB3]

When was the balloon at 400 metres? _____

Explain your reasoning using as much mathematics as you can.

ALGEBRA FORM A RUBRIC

1. AEQB1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Attempt made but uses incorrect weights of bags and balls (i.e., does not use 1 bag = 3 balls)
2	One way drawn correctly
3	Two different ways drawn correctly (e.g., 0 bags, 11 balls; 1 bag, 8 balls or 3 bags, 2 balls)

AEQB2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	One equation written correctly or expression only written (e.g., $3x + 2$)
2	Two equations written correctly (e.g., $2 \text{ bags} + 5 \text{ balls} = 3 \text{ bags} + 2 \text{ balls}$ or $2x + 5 = 3x + 2$)

2. ARLT1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct answer (681) with little or no reasoning (e.g., maybe based on full calculation) or incorrect answer (possibly due to errors in calculation) but recognises the relationship between 347 and 357 (difference of 10)
2	Correct answer with reasoning based on the relationship between 347 and 357 (difference of 10) and the recognition that the missing number must be 10 more than 671

ARELS1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect response (perhaps due to errors in calculation) but with correct reasoning based on the relationship between 521 and 527 (difference of 6)
2	Two correct numbers given (e.g., 13 and 7; 527 and 521) but little/no reasoning.
3	Two correct numbers given where the number on the left is 6 more than the number on the right (e.g., 100 and 94) with reasoning that reflects the relationship between 521 and 527 (difference of 6).

ARELS2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	A different and correct pair.

ARELS3

SCORE	DESCRIPTION
0	No response or irrelevant response

1	Incorrect attempt at describing may be based on previous answers (e.g., <i>add 2 more to both</i>).
2	Statement regarding the difference of 6 (e.g., <i>number on the left must be six more than the number on the right</i>) or expression showing the difference (e.g., $a + 6$, and a)

ARELS7

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Specific solution provided (e.g., <i>c must be 7 and d must be 1 to make it a true number sentence</i>) or a general statement (e.g., <i>c is bigger than d</i>)
2	Statement correctly describes relationship (e.g., <i>c is 7 times the number d</i>)

3. ALEM1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (9 or 9 cups of sugar)

ALEM2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Partially correct response (e.g., ratios 1:4 (sugar:lemons) and 1:3 (water:lemons))
2	Correct response ($\frac{1}{4}$ and $\frac{1}{3}$ or <i>For every lemon you need one quarter cup of sugar and one third cup of water</i>), working shows use of ratios and/or fractions.

ALEM3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Partially correct response (e.g., ratio 1:4) or correct worded explanation but no rule.
2	Correct response (e.g., $\text{sugar} = \frac{1}{4}n$ or <i>For every cup of sugar you need 4 times the amount of lemons, so the amount of sugar needed is one quarter the number of lemons</i>).

4. ACONS1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (Yes)

ACONS2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Reasoning based on examples (e.g., $15 + 16 + 17 = 48$ and $16 \times 3 = 48$)
2	Reasoning based on relationship between the three numbers (e.g. <i>first number is one less than the middle number and third number is one more so if I remove one from the third number and add it to the first all three numbers will be the same or If numbers are $n - 1$, n, $n + 1$, add them together and you get $3n$ and $3n$ divided by 3 is n).</i>

5. ATRNS1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	At least two entries correct
2	Table completed correctly (20,26,32,38)

ATRNS2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (92) with no explanation/working or incorrect response with working to show some understanding of pattern or incorrect with working to show minor calculation error
2	Correct response with an explanation that reflects the use of an additive strategy (e.g., <i>goes up by 6</i> or <i>continues table to a train size of 15</i>)
3	Correct response with an explanation of a multiplicative approach expressed in words or as a rule but not in simplest form (e.g., <i>you multiply 6 by 14 and add 8</i> or <i>working to show $6 \times 14 + 8$</i>)
4	Correct response with an explanation of a multiplicative approach expressed in words or as a rule in simplest form that recognises the 6 wheels in the engine (e.g., <i>you need to times 15 by 6 and add 2</i> or $15 \times 6 + 2$)

ATRNS3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (No) but with no explanation OR incorrect response with sound reasoning
2	Correct response with reasoning to support conclusion (e.g., <i>60 – 2 is 58 and 58 is not divisible by 6 or a size 9 train would have 56 wheels and a size 10 train would have 62 wheels so you can't have a train with 60 wheels.</i>)

6. ATILP1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Calculation based on numbers (e.g., assumes $a = 5$ or 6 cm) that shows an understanding of perimeter (e.g., $5 + 1 + 5 + 1 = 11$ or $12 + 2 = 14$)
2	Correct response but not in its simplest form (e.g., $a + a + 1 + 1$)
3	Correct, simplified response ($2a + 2$ or $2(a + 1)$)

ATILP2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect, but attempt made to solve for the perimeter using symbols (e.g. $a + a + 3 + 3$; $4a + 3$) or assumes $a = 5$ or 6 cm (e.g., $10 + 3 + 10 + 3$ or $12 + 12 + 3 + 3$)
2	Correct response but not in its simplest form (e.g., $2a + 2a + 3 + 3$)
3	Correct symbolic response in simplified form (e.g., $4a + 6$ or $2(2a + 3)$)

ATILP3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but partially identifiable (e.g., $6a + 6$), with little/no working or explanation to support response or incorrect calculated response based on $a = 5$ or 6 cm
2	Incorrect response due to minor errors but with working that shows understanding of perimeter
3	Correct response but not simplified (e.g., $a + 1 + 1 + a + 1 + a + 1 + a - 3 + a - 1 + a + 1 + 2 + 1$) and without clear explanation
4	Correct response ($6a + 4$) without clear explanation (e.g., just added all the sides together) OR correct response but not simplified with a reasonable explanation/working to support solution
5	Correct response ($6a + 4$) with clear explanation for sides that are less than a (e.g., $a - 1$ or $a - 3$) or explanation based on visualisation.

7. AHAB1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Axes labelled correctly but graph not drawn or graph drawn but axes incomplete
2	Partially correct graph with axes labelled correctly (e.g., bar graph or points drawn but not connected) or one or two points plotted incorrectly
3	Line graph and axes drawn and labelled correctly

AHAB2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response ($1\frac{1}{2}$ hours or 1 hour 30 min)

AHAB3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	General response (e.g., <i>look at the graph</i>) or specific time(s) provided but outside range below
2	One correct response, between 4:05 and 4:15 or between 4:35 and 4:45 with little or no explanation
3	Correct response (between 4:05 and 4:15 and between 4:35 and 4:45) but with little or no explanation
4	Correct response with explanation (e.g., find 400m on the graph and look at where the graph crosses the 400 metre mark to determine the time, at 4:00 the balloon reached 350m and at 4:30 the balloon reached 500m. Therefore, the balloon hit 400m in between these two times at around 4:10 or 4:15 and again somewhere between 4:35 and 4:45)

STUDENT SCORE SHEET ALGEBRA FORM A

Student Name:	Year Level:
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		Score	Comments
1	AEQB1		
	AEQB2		
2	ARLT1		
	ARELS1		
	ARELS2		
	ARELS3		
	ARELS7		
3	ALEM1		
	ALEM2		
	ALEM3		
4	ACONS1		
	ACONS2		
5	ATRNS1		
	ATRNS2		
	ATRNS3		
6	ATILP1		
	ATILP2		
	ATILP3		
7	AHAB1		
	AHAB2		
	AHAB3		
Total Raw Score			

RAW SCORE TRANSLATOR FOR ALGEBRA FORM A

The following table locates students on the **Learning Progression for Algebraic Reasoning** based on their total score for Algebra Form A. Total scores are obtained by adding the rubric scores assigned to each item on the form. Where a total score is very close to the beginning or end of a score range, teachers are advised to use their knowledge of the student to make a decision about the most appropriate Zone.

Students need to have had an opportunity to attempt all tasks for this process to be meaningful.

Total Score	Zone	Zone Description
48-49	8	Is able to combine a facility with symbolic representation and an understanding of algebraic concepts to represent and explain mathematical situations. Explanations are sophisticated using logical thought and the language of reasoning. Can use multiple representations in a co-ordinated manner to solve, analyse, convince and conclude. Can visualise the form and structure of a function, at least graphically, from a real context. Is able to work in a context free environment using symbolic language and treat algebraic expressions (e.g. $3x + 2$) as single entities. Can generalise more complex situations. Is able to establish and describe equivalence involving the four operations explaining relationships in symbolic terms. Can use abstract symbols to solve problems in context with multiple steps.
43-47	7	Is able to use and interpret algebraic conventions for representing generality and relationships between variables. Beginning to use sound logical reasoning with appropriate reasoning language (e.g. if ... then, must) evident. There is more co-ordination of multiplicative thinking and the associated language to notice algebraic structure. Can recognise and use the relationships between multiple entities and connections between and within different representations. Is able to establish and describe equivalence explaining relationships using the distributive property and the inverses of addition and multiplication. Can generalise quite complex situations and in more direct situations is beginning to use simplest form.
37-43	6	Can use and interpret basic algebraic conventions to represent situations involving a variable quantity. Beginning to explain using logical language and to use if ... then reasoning. Uses symbolic language but the need for simplification is still being developed. Able to generalise arithmetic relationships with justification, including simple multiplicative relationships, but are often still context bound. Can show why several expressions are equivalent, typically employing numerical (non-symbolic) justifications.
30-36	5	Able to use multiplicative reasoning in simple situations. Can reason with more complex additive situations involving larger numbers and subtraction but usually by examples. Has moved from algebraic expressions to using equations. Can derive a strategy that maintains equivalence, but cannot

		yet generalise the situation. Able to use symbols to express rules. Can follow, compare and explain rules for linking successive terms in a sequence. Recognises and represents simple functional representations. Can justify an argument using mathematical text. Beginning to generalise using words or using some symbolic generalisations in simple situations, usually building on in context.
28-29	4	Beginning to work multiplicatively and simultaneously co-ordinate variables, although still uses specific examples to convince. Able to reason and generalise in simple situations. Can recognise and interpret the relevance of range from table and/or graphs and to recognise functional relationships. When faced with more complex algebraic situations is unable to use the full range of explanation or handle all of the information simultaneously. Beginning to transition to abstraction by inserting a number for a pronumeral.
16-27	3	Beginning to use symbolic expression and elementary reasoning. While still using arithmetic approaches there is evidence of relational reasoning with the numbers and providing some explanation. Beginning to recognise simple multiplicative relationships. There is some evidence of co-ordination of two ideas. Explanation and justification is limited. Algebraic expressions are used rather than equations. Beginning to recognise equivalent relationships. Can explain simple generalisations by telling stories, manipulating materials and very simple use of symbolic language.
8-15	2	Beginning to recognise patterns and relationships and conjecture about these. Able to identify numbers that vary and numbers that stay the same. Engages with the context, but arithmetic reasoning, typically based on calculations, is still being used. Recognises some multiples and some relationships like 6 more/6 less, while not necessarily recognising equivalence. Can work with simple scales and transfer from a table of values to a graph.
1-7	1	Can continue simple patterns but is likely to build them additively. Reasoning is confined to specific incidences and numerical examples of simple physical situations. Arithmetic thinking is used. Abstraction and generalisation not evident at this stage.