<table>
<tr><th>Zone Description</th><th>Teaching Implications</th></tr>
<tr><td rowspan="2">Zone 1 – Primitive Modelling

Can solve simple multiplication and division problems involving relatively small whole numbers (e.g. Butterfly House parts a and b)*, but tends to rely on drawing, models and count-all strategies (e.g. draws and counts all pots for part a of Packing Pots). May use skip counting (repeated addition) for groups less than 5 (e.g. to find number of tables needed to seat up to 20 people in Tables and Chairs)

Can make simple observations from data given in a task (e.g. Adventure Camp a) and reproduce a simple pattern (e.g. Tables and Chairs a to e)

Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation

* these items are from the original SNMY research</td>
<td>Consolidate/establish:

Trusting the count for numbers to 10 (e.g. for 6 this involves working with mental objects for 6 without having to model and/or count-all). Use flash cards to develop subitising (i.e. ability to say how many without counting) for numbers to 5 initially and then to 10 and beyond using part-part-whole knowledge (e.g. 8 is 4 and 4, or 5 and 3 more, or 2 less than 10). Practice regularly

Simple skip counting to determine how many in a collection and to establish numbers up to 5 as countable objects, for example, count by twos, fives and tens, using concrete materials and a 0-99 Number Chart

Mental strategies for addition and subtraction facts to 20 for example, Count on from larger (e.g. for 2 and 7, think, 7, 8, 9), Double and near doubles (e.g. use ten-frames and a 2-row bead-frame to show that 7 and 7 is 10 and 4 more, 14), and Make-to-ten (e.g. for 6 and 8, think, 8, 10, 14, scaffold using open number lines). Explore and name mental strategies to solve subtraction problems such as 7 take 2, 12 take 5, and 16 take 9. Practice (e.g. by using Number Charts from Maths 300)

2-digit place-value – working flexibly with ones and tens by making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming. Play the 'Place-Value Game' (see Siemon et al, 2015)[3]</td></tr>
<tr><td>Introduce/develop:

Doubling (and halving) strategies for 2-digit numbers that do not require renaming (e.g. 34 and 34, half of 46), build to numbers that require some additional thinking (e.g. to double 36, double 3 tens, double 6 ones, 60 and 12 ones, 72)

Extended mental strategies for addition and subtraction, use efficient, place-value based strategies (e.g. 37 and 24, think: 37, 47, 57, 60, 61). Use open number lines to scaffold thinking

Efficient and reliable strategies for counting large collections (e.g. count a collection of 50 or more by 2s, 5s or 10s) with a focus on how to organise the number of groups to facilitate the count (e.g. by arranging the groups systematically in lines or arrays and then skip counting)

How to make, name and use arrays/regions to solve simple multiplication or sharing problems using concrete materials, and skip counting (e.g. 1 four, 2 fours, 3 fours …), leading to more efficient counting strategies based on reading arrays in terms of a consistent number of rows (e.g. 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours, …)

3-digit place-value – working flexibly with tens and hundreds (by making with MAB, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming - see Siemon et al, 2015)

Strategies for unpacking and comprehending problem situations (e.g. read and re-tell, ask questions such as, What is the question asking? What do we need to do? …). Use realistic word problems to explore different ideas for multiplication and division. For example, 3 rows, 7 chairs in each row, how many chairs (array)? Mandy has three times as many…as Tom…, how many … does she have (scalar idea)? 24 cards shared among 6 students, how many each (partition)? Lollipops cost 5c each, how much for 4 ('for each' idea)?

How to explain and justify solution strategies orally and in writing through words and pictures (important for mathematical literacy)</td></tr>
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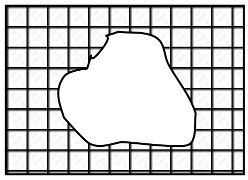
[3] Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R. & Warren, E. (2015). *Teaching Mathematics: Foundations to Middle Years.* Melbourne: Oxford University Press

Zone 2: Intuitive Modelling	**Consolidate/establish:**
Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting (e.g. *Tables & Chairs j, Butterfly House d*), counts large collections efficiently, systematically keeps track of count (for instance may order groups in arrays or as a list) but needs to 'see' all groups (e.g. *Tiles, Tiles, Tiles a*, or for *Butterfly House e*, may use list and/or doubling as follows: 2 butterflies 5 drops 4 butterflies 10 drops 6 butterflies 15 drops … 12 butterflies 30 drops) Can share collections into equal groups/parts (e.g. *Pizza Party a* and *b*). Recognises small numbers as composite units (e.g. can count equal groups, skip count by twos, threes and fives) Recognises multiplication is relevant (e.g. *Packing Pots c, Speedy Snail a*) but tends not to be able to follow this through to solution Can list some of the options in simple Cartesian Product situations (e.g. *Canteen Capers a*) Orders 2-digit numbers (e.g. partially correct ordering of times in *Swimming Sports a*) Some evidence of multiplicative thinking as equal groups/shares seen as entities that can be counted systematically	See the **Ideas and strategies** introduced/developed in the previous Zone **Introduce/develop:** **More efficient strategies for counting groups** based on a **change in focus** from a count of equal groups (e.g. (1 three, 2 threes, 3 threes, 4 threes, …) to a consistent number of groups (e.g. 3 ones, 3 twos, 3 threes, 3 fours, …) which underpin the more efficient mental strategies listed below and ultimately lead to the **factor-factor-product idea** **Array/region-based mental strategies for multiplication facts to 100** for example, *doubling* (for 2s facts), *doubling and 1 more group* (for 3s facts)*, double doubles* (for 4s facts)*, relate to tens* (for 5s and 9s facts) and so on (see *There's More to counting Than Meets the Eye*) Efficient strategies for solving problems where **arrays** and **regions** only partially observed  For example, paint is spilled on a tiled floor. How many tiles to replace? How many altogether? How do you know? **Commutativity,** by exploring the relationship between arrays and regions such as 3 fours and 4 threes. Play 'Multiplication Toss' **Informal division strategies** such as *think of multiplication* and *halving,* (e.g. 16 divided by 4, think: 4 'whats' are 16? 4; or half of 16 is 8, half of 8 is 4) **Extended mental strategies for multiplication** (e.g. for 3 twenty-fives, Think: double 25, 50, and twenty-five more, 75) and use place-value based strategies such as 10 groups and 4 more groups for 14 groups **Simple proportion problems** involving non-numerical comparisons (e.g. If Nick mixed less cordial with more water than he did yesterday, his drink would taste (a) stronger, (b) weaker (c) exactly the same, or (d) not enough information to tell) **How to recognise and describe simple relationships** and patterns (e.g. 'double and add 2' from models, diagrams and tables; or notice that a diagonal pattern on a 0-99 chart is a count of 11, 1 ten and 1 ones) **Language of fractions** through practical experience with both **continuous and discrete, 'real-world' fraction models** for example, 3 quarters of the pizza, half the class), distinguish between how many and how much (e.g. in 2 thirds the numeral indicates how many, the name indicates how much) **Halving partitioning strategy**, through paper folding (kinder squares and streamers), cutting plasticine 'cakes' and 'pizzas', sharing collections equally (counters, cards etc), apply thinking involved to help children create their own fraction diagrams. Focus on making and naming parts in the halving family (e.g. 8 parts, eighths) including mixed fractions (e.g. "2 and 3 quarters") and informal recording (e.g. 3 eighths), no symbols **Key fraction generalisations** – that is, that equal parts are necessary and that the number of parts names the part

Zone 3: Sensing	**Consolidate/establish:**
Demonstrates intuitive sense of proportion (e.g. partial solution to *Butterfly House f*) and partitioning (e.g. *Missing Numbers b*) Works with ‘useful’ numbers such as 2 and 5, and strategies such as doubling and halving (e.g. *Packing Pots b,* and *Pizza Party c*) May list all options in a simple Cartesian product situation (e.g. *Canteen Capers b*), but cannot explain or justify solutions Uses abbreviated methods for counting groups, for example, doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions (e.g. *Pizza Party c*) Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems (e.g. *Stained Glass Windows a* and *b, Tiles, Tiles, Tiles b)*	See the **Ideas and strategies** introduced/developed in the previous Zone **Introduce/develop:** **Place-value based strategies** for informally solving problems involving single-digit by two-digit multiplication (e.g. for 3 twenty-eights, THINK, 3 by 2 tens, 60 and 24 more, 84) mentally or in writing Initial **recording to support place-value** for multiplication facts (see *Siemon et al, 2015* and *There’s More to Counting Than Meets the Eye*) More **efficient strategies for solving number problems involving simple proportion** (e.g. recognise as two-step problems, What do I do first? Find value for common amount. What do I do next? Determine multiplier/factor and apply. Why?) How **to rename number of groups** (e.g. think of 6 fours as 5 fours and 1 more four), Practice (e.g. by using ‘Multiplication Toss Game’*).* Re-name composite numbers in terms of equal groups (e.g. 18 is 2 nines, 9 twos, 3 sixes, 6 threes) **Cartesian product** or **for each** idea using concrete materials and relatively simple problems such as 3 tops and 2 bottoms, how many outfits, or how many different types of pizzas given choice of small, large, medium and 4 varieties? Discuss how to recognise problems of this type and how to keep track of the count such as draw all options, make a list or a table (tree diagrams appear to be too difficult at this level, these are included in Zone 5) How to **interpret problem situations and solutions relevant to context** (e.g. Ask, What operation is needed? Why? What does it mean in terms of original question?) Simple, practical division problems that require the **interpretation of remainders** relevant to context **Practical sharing situations that introduce names for simple fractional parts beyond the halving family** (e.g. thirds for 3 equal parts/shares, sixths for 6 equal parts etc) and help **build a sense of fractional parts**, for example, 3 sixths is the same as a half or 50%, 7 eighths is nearly 1, “2 and 1 tenth” is close to 2. Use a range of continuous and discrete fraction models including mixed fraction models **Thirding and fifthing partitioning strategies** through paper folding (kinder squares and streamers), cutting plasticine ‘cakes’ and ‘pizzas’, sharing collections equally (counters, cards etc), **apply thinking involved to help children create their own fraction diagrams (regions) and number line representations** (see Siemon (2004) *Partitioning – The Missing Link in building Fraction Knowledge and Confidence*. Focus on making and naming parts in the thirding and fifthing families (e.g. 5 parts, fifths) including mixed fractions (e.g. “2 and 5 ninths”) and informal recording (e.g. 4 fifths), no symbols. Revisit key fraction generalisations (see Level 2), include whole to part models (e.g. partition to show 3 quarters) and part to whole (e.g. if this is 1 third, show me the whole) and use diagrams and representations to **rename related fractions** **Extend partitioning strategies** to construct number line representations. Use multiple fraction representations **Key fraction generalisations** – equal parts, as the number of parts increase the size of the part gets smaller; the number of parts names the part (e.g., 8 parts, eighths) and the size of the part depends upon the size of the whole

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<td rowspan="2">Zone 4: Strategy Exploring

Solves more familiar multiplication and division problems involving two-digit numbers (e.g. Butterfly House c and d, Packing Pots c, Speedy Snail a)

Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations (e.g. Packing Pots d, Filling the Buses a and b, Tables & Chairs g and h, Butterfly House h and g, Speedy Snail c, Computer Game a, Stained Glass Windows a and b). Tend not to explain their thinking or indicate working

Able to partition given number or quantity into equal parts and describe part formally (e.g. Pizza Party a and b), and locate familiar fractions (e.g. Missing Numbers a)

Beginning to work with simple proportion, for example, can make a start, represent problem, but unable to complete successfully or justify their thinking (e.g. How Far a, School Fair a and b)</td>
<td>Consolidate/establish:

See the Ideas and strategies introduced/developed in the previous Zone</td>
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<td>Introduce/develop:

More efficient strategies for multiplying and dividing larger whole numbers independently of models (e.g. strategies based on: doubling, renaming the number of groups, factors, place-value, and known addition facts,

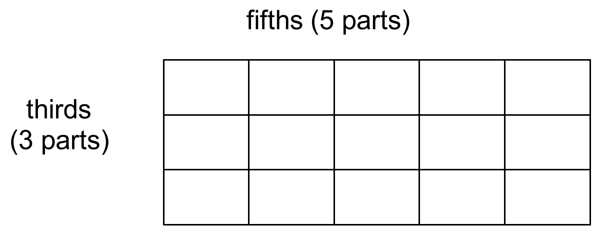
for example, for dividing 564 by 8, THINK, 8 what's are 560? 8 by 7 tens or 70, so 70 and 4 remainder.

for example, for 3908 divided by 10, RENAME as, 390 tens and 8 ones, so 390.8)

Tenths as a new place-value part, by making/representing, naming and recording ones and tenths (see Siemon et al, 2015), consolidate by comparing, ordering, sequencing counting forwards and backwards in ones and/or tenths, and renaming

How to partition continuous quantities more generally using the halving, thirding, fifthing strategies (see Siemon et al, 2015 and Siemon (2004) Partitioning – The Missing Link in building Fraction Knowledge and Confidence), for example, recognise that sixths can be made by halving and thirding (or vice versa), tenths can be made by fifthing and halving etc, use this knowledge to construct fraction diagrams (e.g. region models) and representations (e.g. number line) for common fractions and decimals including mixed numbers

Informal, partition-based strategies for renaming simple unlike fractions, for example, recognise that thirds and fifths can be renamed by thirding and then fifthing (or vice versa) on a common diagram, for example,



Link to region model of multiplication (in this case 3 fives, or 3 parts by 5 parts) to recognise that thirds by fifths are fifteenths, so 2 thirds (2 rows) can be renamed as 10 fifths and 4 fifths (4 columns) can be renamed as 12 fifteenths. Use partitioning strategies to informally add and subtract like and related fractions

Key fraction generalisations - that is, recognise that equal parts are necessary, the total number of parts names the part, and as the total number of parts increases they get smaller (this idea is crucial for the later development of more formal strategies for renaming fractions (see Level 5) which relate the number of parts initially (3, thirds) to the final number of parts (15, fifteenths) in terms of factors, that is, the number of parts has been increased by a factor of 5)

Metacognitive strategies to support problem comprehension, problem representation, strategy monitoring/checking, and interpretation of outcomes relevant to context (see Siemon and Booker (1990) paper on Teaching and Learning For, About and Through Problem Solving)

Simple proportion problems that introduce techniques for dealing with these situations (e.g. find for 1 then multiply or divide as appropriate, using scale diagrams and interpreting distances from maps)</td>
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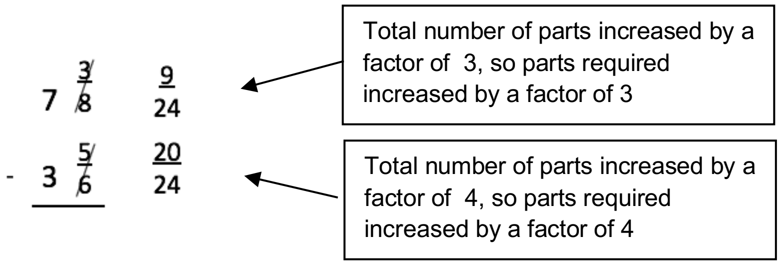
Zone 5: Strategy Refining	**Consolidate/establish:**
Systematically solves simple proportion and array problems (e.g. *Butterfly House e*, *Packing Pots a, How Far a*) suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions (e.g. *School Fair a, Speedy Snail b*) Able to solve simple, 2-step problems using a recognised rule/relationship (e.g. *Fencing the Freeway a*) but finds this difficult for larger numbers (e.g. *Tables & Chairs* k and *l, Tiles, Tiles, Tiles c, Stained Glass Windows c*) Able to order numbers involving tens, ones, tenths and hundredths in supportive context (*Swimming Sports a*) Able to determine all options in Cartesian product situations involving relatively small numbers, but tends to do this additively (e.g. *Canteen Capers* a, *Butterfly House l* and *i*) Beginning to work with decimal numbers and percent (e.g. *Swimming Sports a* and *b*, *Computer Game b*) but unable to apply efficiently to solve problems Some evidence that multiplicative thinking being used to support partitioning (e.g. *Missing Numbers b*) Beginning to approach a broader range of multiplicative situations more systematically	See the **Ideas and strategies** introduced/developed in the previous Zone
	Introduce/develop: **Place-value** ideas and strategies for 5 digits and beyond if not already developed and decimal fractions to hundredths (see partitioning below) including renaming **Flexible, meaningful and efficient strategies** for multiplying and dividing by multiples of ten (e.g. 2.13 by 10, THINK, 21 ones and 3 tenths, 21.3) The **area idea** to support multi-digit multiplication and formal recording (see Siemon et al, 2015) and more efficient strategies for representing and solving an **expanded range of Cartesian product problems** involving three or more variables and **tree diagram representations** **Formal terminology** associated with multiplication and division such as factor, product, divisor, multiplier and raised to the power of …. Play 'Factor Cross Game'. Use calculators to explore what happens with repeated factors, for example, 4 x 4 x 4 x 4 …, factors less than 1, and negative factors. **Informal, partition-based strategies for renaming an expanded range of unrelated fractions** as a precursor to developing an efficient, more formal strategy for generating equivalent fractions (see below), for example, explore using paper folding, diagrams and line models how sixths and eighths could be renamed as forty-eighths but they can also be renamed as twenty-fourths because both are factors of 24 The **generalisation for renaming fractions**, that is, if the number of equal parts (represented by the denominator) increases/decreases by a certain factor then the number of parts required (indicated by the numerator) increases/decreases by the same factor **Written solution strategies for the addition and subtraction of unlike fractions**, for example, think of a diagram showing sixths by eighths … forty-eighths… Is this the simplest? No, twenty-fourths will do, rename fractions by inspection  9 twenty-fourths can't take 20 twenty-fourths, trade 1 one for 24 twenty-fourths to get 6 and 33 twenty-fourths, subtraction is then relatively straightforward Explore **link between multiplication and division and fractions** including decimals (e.g. 3 pizzas shared among 4, 3 divided by 4 is 0.75 etc) to understand **fraction as operator idea** (e.g. ¾ of 120, 75% of $48, 250% of 458,239). Use 'Multiple Patterns Worksheet' (See Support Materials). Establish benchmark equivalences (e.g. 1 third =$33\frac{1}{3}$ %) **Metacognitive strategies** to support problem comprehension, strategy monitoring/checking, and interpretation of outcomes relevant to context (see Siemon and Booker (1990) paper on *Teaching and Learning For, About and Through Problem Solving*)

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Zone 6: Strategy Extending

Can work with Cartesian Product idea to systematically list or determine the number of options (e.g. *Canteen Capers b*, *Butterfly House i* and *h*)

Can solve a broader range of multiplication and division problems involving two digit numbers, patterns and/or proportion (e.g. *Tables & Chairs h*, *Butterfly House f, Stained Glass Windows b* and *c, Computer Game a and b*) but may not be able to explain or justify solution strategy (e.g. *Fencing the Freeway b, Fencing the Freeway d,* and *Swimming Sports b, How Far b, Speedy Snail b*)

Able to rename and compare fractions in the halving family (e.g. *Pizza Party* c) and use partitioning strategies to locate simple fractions (e.g. *Missing Numbers* a)

Developing sense of proportion (e.g. sees relevance of proportion in *Adventure Camp b*, *Tiles, Tiles, Tiles b*), but unable to explain or justify thinking

Developing a degree of comfort with working mentally with multiplication and division facts

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Consolidate/establish:

See the **Ideas and strategies** introduced/developed in the previous Zone

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Introduce/develop:

Hundredths as a new place-value part, by making/representing, naming and recording ones, tenths, and hundredths (see Siemon et al, 2015), consolidate by comparing, ordering, sequencing counting forwards and backwards in place-value parts, and renaming. Link to %

How to **explain and justify solution strategies** for problems involving multiplication and division (see Multiplication Workshop handout in Support Materials), particularly in relation to interpreting decimal remainders appropriate to context, for example,

> How many buses will be needed to take 594 students and teachers to the school Speech night, assuming each bus hold 45 passengers and everyone must wear a seatbelt?

More **efficient, systematic, and/or generalizable processes** for dealing with **proportion problems** (e.g. use of the 'for each' idea, formal recording, and the use of fractions, percent to justify claims), for example,

> Jane scored 14 goals from 20 attempts. Emma scored 18 goals from 25 attempts. Which girl should be selected for the school basketball team and why?
>
> 6 girls share 4 pizzas equally. 8 boys share 6 pizzas equally. Who had more pizza, the girls or the boys?
>
> 35 feral cats were found in a 146 hectare nature reserve. 27 feral cats were found in a 103 hectare reserve. Which reserve had the biggest feral cat problem?
>
> Orange juice is sold in different sized containers: 5L for $14, 2 L for $5, and 500mL for $1.35. Which represents the best value for money?

More efficient strategies and formal processes for working with multiplication and division involving larger numbers based on sound place-value ideas, for example, 3486 x 21 can be estimated by thinking about 35 hundreds by 2 tens, 70 thousands, and 1 more group of 35 hundred, that is, 73,500, or it can be calculated by using factors of 21, that is, 3486 x 3 x 7. Two-digit multiplication can be used to support the multiplication of ones and tenths by ones and tenths, for example, for 2.3 by 5.7, rename as tenths and compute as 23 tenths by 57 tenths, which gives 1311 hundredths hence 13.11. Consider a broader range of problems and applications, for example,

> Average gate takings per day over the World Cricket cup Series
>
> Matt rode around the park 8 times. The odometer on his bike indicated that he ridden a total of 15 km. How far was it around the park?
>
> After 11 training sessions, Kate's average time for 100 metres butterfly was 61.3 seconds. In her next 2 trials, Kate clocked 61.21 and 60.87 seconds. What was her new average time?

Integers using real-world examples such as heights above and below sea-level, temperatures above and below zero, simple addition and difference calculations

The **notion of variable** and how to **recognise and formally describe patterns** involving all four operations. Use 'Max's Matchsticks' to explore how patterns may be viewed differently leading to different ways of counting and forms of representation.

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Zoom 7: Connecting	Consolidate/establish:
Zone 7: Connecting Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording (e.g. *Filling the Buses a, Fencing the Freeway d, Packing Pots d*) Can solve and explain solutions to problems involving simple patterns, percent and proportion (e.g. *Fencing the Freeway c, Swimming Sports b, Butterfly House g, Tables & Chairs g* and *l, Speedy Snail c, Tiles, Tiles, Tiles b and c, School Fair a, Stained Glass Windows a, Computer Game b, How Far b*). May not be able to show working and/or explain strategies for situations involving larger numbers (e.g. *Tables & Chairs m* and *k, Tiles, Tiles, Tiles c*) or less familiar problems (e.g. *Adventure Camp b, School Fair b, How Far c*) Locates fractions using efficient partitioning strategies (e.g. *Missing Numbers a*) Beginning to make connections between problems and solution strategies and how to communicate this mathematically	**Consolidate/establish:** See the **Ideas and strategies** introduced/developed in the previous Zone **Introduce/develop:** **Strategies for comparing, ordering, sequencing, counting forwards and backwards in place-value parts, and renaming** large whole numbers, common fractions, decimals, and integers (e.g. a 3 to 4 metre length of rope, appropriately labelled number cards and pegs could be used to sequence numbers from 100 to 1,000,000, from -3 to +3, from 2 to 5 and so on). The metaphor of a magnifying glass can be used to locate numbers involving hundredths or thousandths on a number line as a result of successive *tenthing* (see Siemon et al, 2015 and Siemon (2004) *Partitioning – The Missing Link in building Fraction Knowledge and Confidence*) An appreciation of **inverse and identity relations,** for example, recognise which number when added leaves the original number unchanged (zero) and how inverses are determined in relation to this, for example, the inverse of 8 is -8 as -8 + 8 = 0 and 8 + -8 = 0. In a similar fashion, recognise that 1 is the corresponding number for multiplication, where the inverse of a number is defined as its **reciprocal**, for example, the inverse of 8 is $\frac{1}{8}$ **Index notation** for representing multiplication of repeated factors, for example, $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$ A **more generalised understanding of place-value** and the structure of the number system in terms of exponentiation, for example, $10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2\ \ 10^3 \ldots$ **Strategies to recognise and apply** multiplication and division in a broader range of situations including ratio, proportion, and unfamiliar, multiple-step problems, for example, *Orange Juice* task (see Support Materials) How to **recognise and describe number patterns more formally** for example, triangular numbers, square numbers, growth patterns (e.g. 'Garden Beds' from *Maths 300* and 'Super Market Packer' from Support Materials) **Notation to support general arithmetic** (simple algebra), for example, recognise and understand the meaning of expressions such as $x+4$, $3x$, $5x^2$, or $\frac{x-1}{3}$ **Ratio as the comparison of any two quantities**, for example, the comparison of the number of feral cats to the size of the national park. Recognise that ratios can be used to compare measures of the same type (e.g. the number of feral cats compared to the number of feral dogs) and that within this, two types of comparison are possible, for instance, one can compare the parts to the parts (e.g. cats to dogs) or the parts to the whole (e.g. cats to the total number of cats and dogs). Ratios can be also used to compare measures of different types, when used this way they are referred to as rates (e.g. the number of feral cats per square kilometre). Ratios are not always rational numbers (e.g. the ratio of the circumference of a circle to its diameter) **Strategies for recognising and representing proportion problems involving larger numbers and/or fractions** (e.g. problems involving scale such as map calculations, increasing/reducing ingredients in a recipe, and simple problems involving derived measures such as volume, density, speed, and chance)

Zone 8: Reflective Knowing	**Consolidate/establish:** See the **Ideas and strategies** introduced/developed in the previous Zone
Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals (e.g. *Adventure Camp b*, *Speedy Snail b*) Can justify partitioning (e.g. *Missing Numbers b*) Can use and formally describe patterns in terms of general rules (e.g. *Tables and Chairs*, *m* and *k*) Beginning to work more systematically with complex, open-ended problems (e.g. *School Fair b, Computer Game c*)	**Introduce/develop:** **A broader range of multiplicative situations** for example, problems involving the calculation of area or volume, derived measures and rates, variation, complex proportion, and multiple step problems involving large whole numbers, decimals and fractions, for example, Find the volume of a cylinder 4 cm in diameter and 9 cm long. Find the surface area of a compound shape Foreign currency calculations Determine the amount of water lost to evaporation from the Hume Weir during the summer. **Strategies for simplifying expressions** for example, adding and subtracting like terms, and justifying and explaining the use of cancellation techniques for division through the use of common factors, for example, $\frac{42a}{7} = 6a$ because $\frac{42a}{7} = \frac{7 \times 6a}{7}$ and $\frac{7}{7} = 1$ **Algebraic reasoning and representation strategies** to solve problems involving multiplicative relationships, for example, If 2 T-shirts and 2 drinks cost \$44 and 1 T-shirt and 3 drinks cost \$30, what is the price of each? 5 locker keys are returned at random to the students who own them. What is the probability that each student will receive the key that opens their locker? A mad scientist has a collection of beetles and spiders. The sensor in the floor of the enclosure indicated that there were 174 legs and the infra-red image indicated that there were 26 bodies altogether. How many were beetles and how many were spiders? 365 is an extraordinary number. It is the sum of 3 consecutive square numbers and also the sum of the next 2 consecutive square numbers. Find the numbers referred to. **Strategies for working with numbers and operations expressed in exponent form**, for example, why $2^3 \times 2^6 = 2^9$, investigate the structure of the place value system in terms of positive and negative powers of 10 **Explore non-linear, exponential situations** such as growth and decay (e.g. Radioactivity activity from maths300) **Writing mathematically** using appropriate symbolic text, using equivalent sentences to systematically arrive at a solution **More abstract problem solving situations** requiring an appreciation of problem solving as a process, the value of recognising problem type, and the development of a greater range of strategies and representations (e.g. tables, symbolic expressions, rule generation and testing) including the manipulation of symbols