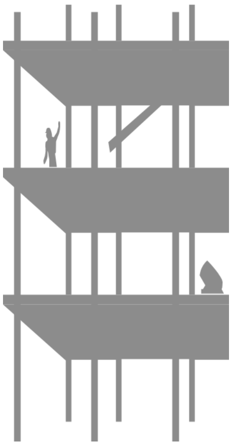


Assessment Materials for Multiplicative Thinking

Assessment Task Booklet
Option 4

GROWING Mathematically

Based on the *Scaffolding Numeracy in the Middle Years* and *Reframing Mathematical Futures II* Research projects

ASSESSING MULTIPLICATIVE THINKING

ASSESSMENT TASK BOOKLET FOR MULTIPLICATIVE THINKING
OPTION 4

NAME
YEAR LEVEL:

This booklet contains an Extended Task and 6 Supplementary or Short Tasks:

X1 – Board Toom Tables
S1 – Butterfly House
S2 – Canteen Capers
S3 – Lemonade
S4 – Hat Chance
S5 – Spy Squad
S6 – Park Map

<u>INSTRUCTIONS:</u>

1. Please do as much of each task as you can. Some tasks you will find easy; others will be more difficult.
2. All working must be shown in this booklet. If you need more space, please use the back of the previous page or another space, but make sure we know where to find your answer.
3. When you are asked to **show all your working and explain your answer in as much detail as possible** or to **explain your reasoning using as much mathematics as you can** or **show all your working so we can understand your thinking** do your best to write down what you did and why, in the space provided.
4. Don't rub out any work that you think is incorrect. Simply draw a line through it.
5. If you have any questions please ask your teacher.

Board Room Tables

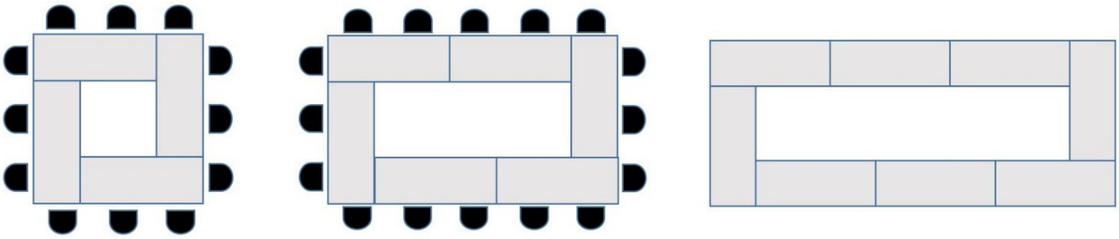
In order to be flexible a Board Room has several tables that can be arranged to cater for different numbers of people at Board meetings.

Each table is a rectangle.

Each table can seat one person on its short edge and two people on its long edge.

The diagrams below show how these tables can be arranged for different numbers of people.

(No one sits inside the arrangement.)



Size 1	Size 2	Size 3
4 tables	6 tables	tables
12 people	16 people	people

[ABRT2]

How many tables are in a Size 4 arrangement? ____________________

[ABRT3]

How many tables are in a Size 7 arrangement? ______________________

Explain your reasoning.

[ABRT4]

Write down in words or symbols a rule for working out the number of tables when you know the Size number.

[ABRT5]

Write down in words or symbols a rule for working out the table Size given the number of tables.

[ABRT6]

John said he needed 13 tables to set up for his meeting. Could John be correct?
Explain how you know.

[ABRT7]

What Size arrangement is needed to seat 72 people?

Explain your reasoning.

[ABRT8]

Write down in words or symbols a rule for working out the Size of the arrangement when you know the number of people.

Butterfly House ... [BTH]

Some children visited the Butterfly House at the Zoo.
They learnt that a butterfly is made up of 4 wings, one body and two feelers.

While they were there, they made models and answered some questions.

For each question, **explain your working and your answer**, in as much detail as possible.

a. How many wings, bodies and feelers would be needed for 7 model butterflies? **Show all your working and explain your answer in as much detail as possible.**

________________ wings

________________ bodies

________________ feelers

b. How many complete model butterflies could you make with 16 wings, 4 bodies and 8 feelers? **Show all your working and explain your answer in as much detail as possible.**

c. How many wings, bodies and feelers will be needed to make 98 model butterflies. **Show all your working and explain your answer in as much detail as possible.**

________________ wings

________________ bodies

________________ feelers

d. How many complete model butterflies could you make with 29 wings, 8 bodies and 13 feelers? **Show all your working and explain your answer in as much detail as possible.**

Canteen Capers … [CC]

The school canteen makes great lunches. They offer 2 choices of rolls, 4 choices of filling and 3 choices of drink.

Rolls	Fillings	Drinks
White	Peanut butter	Orange Juice
Multi-grain	Salad	Apple Juice
	Cheese and vegemite	Cola
	Tuna and lettuce	

For each question, show your working and explain your answer in as much detail as possible.

a. Claire ordered a peanut-butter roll and a drink. What might she have ordered? List all possibilities.

b. There are 23 students in Claire's class. If everyone ordered a roll with a filling and a drink, could they all have a different lunch order?

Lemonade Recipe

[ALEM1]

- 3 cups of sugar,
- 4 cups of water
- Juice of 12 medium sized lemons.

How much sugar would be needed if 12 cups of water were to be used?

[ALEM2]

Explain how much sugar and how much water would be needed for each lemon used.

[ALEM3]

Write a rule for the amount of sugar needed for n lemons, where n represents the number of lemons. Explain your reasoning.

Hat Chance

[SHAT8]

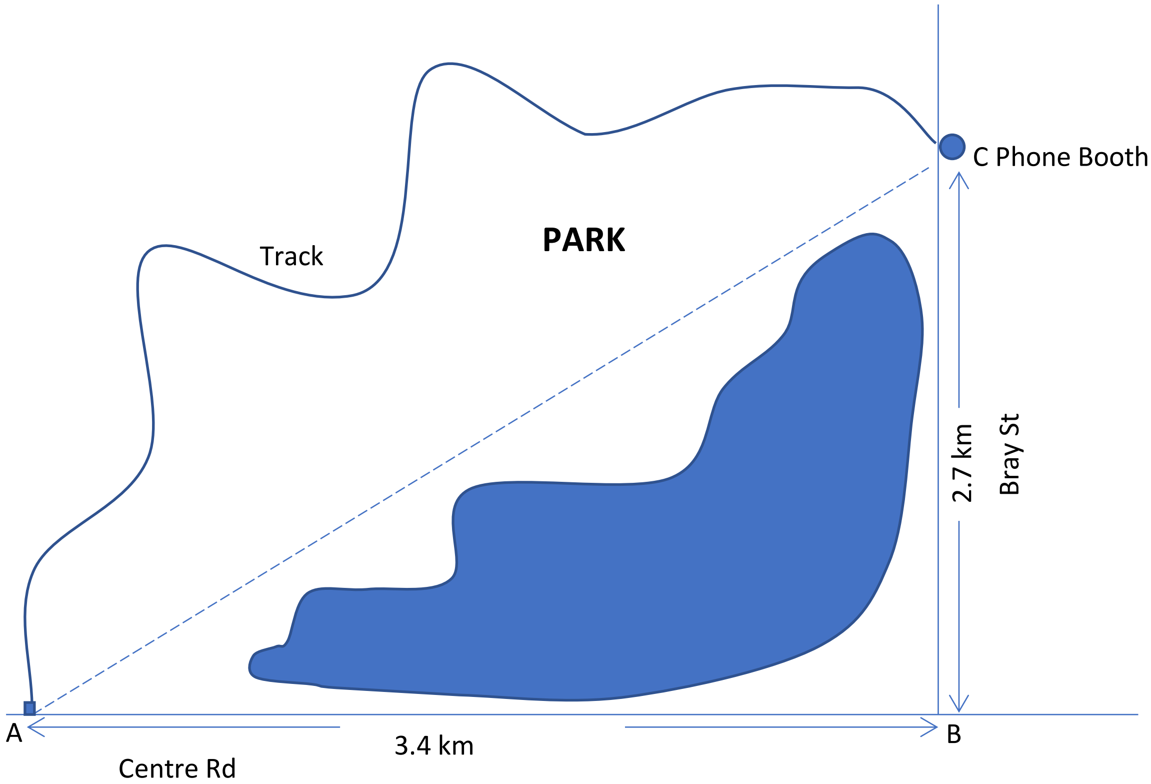
A mathematics class has 13 boys and 16 girls in it. Each pupil's name is written on a piece of paper. All the names are put in a hat. The teacher picks out one name without looking. Tick the box to show which outcome is more likely

- ☐ the name is a boy or
- ☐ the name is a girl or
- ☐ the name could be a boy or a girl

Please explain your answer **using as much mathematics as you can**.

Spy Squad

Your squad is being sent to a phone booth on Bray St to pick up a clue. You are in a taxi going along Centre Rd but the traffic is bad so the average speed is only 8 km per hour. You are following your progress on the map below, but the taxi is currently stopped at point A on the map.



[GSPSQ7]

How long will it take the taxi to get to point B if the speed is 8 km per hour?

[GSPSQ8]

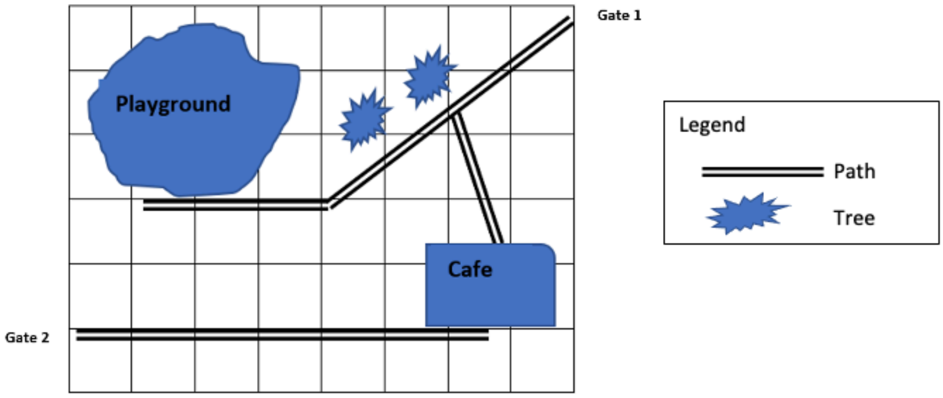
How long will it take the taxi to travel from B to the phone booth at C if the average speed is still 8 km per hour?

[GSPSQ9]

You know that your running speed over a distance is 6 km per hour. The track through the park is winding so would not help but the main part of the park is lawn so you could run across the lawn (along the dotted line you marked on the map). How long would that take you and would you be faster to get out of the taxi and run or stay in the car? Explain how you decided.

Park Map

This is the map of a park. Answer the questions below as accurately as you can.



Scale: Each square represents 200m by 200m

[GMAPA] What is the approximate distance along the path from the café to Gate 2?

[GMAPA1] Explain how you got your answer.

[GMAPB] If I walked in the park from Gate 1 to the playground following the path, approximately how far would I walk?

[GMAPB1] Explain how you got your answer.

[GMAPC] Sam wanted to run in a straight line from the corner of the café closest to the playground to the playground in the park. Approximately how far is that distance?

[GMAPC1] Explain how you got your answer.

[GMAPD] Explain how you might find any distance in the park.

ASSESSING MULTIPLICATIVE THINKING

SCORING RUBRIC OPTION 4

BOARD ROOM TABLES [ABRT2]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (10 or 10 tables)

BOARD ROOM TABLES [ABRT3]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (16 tables) with no explanation or evidence of additive thinking (e.g., continues table or *it goes up by two* or *add two each time*)
2	Correct response with reasonable explanation either in words (e.g., *The number of tables along each long side of the arrangement is the same as the Size number, so you multiply this by two and add the two tables, one for each end*) or in symbols (N = 2S + 2 or 2(S + 1)).

BOARD ROOM TABLES [ABRT4]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Response suggests additive thinking (e.g., *goes up by two* or *add two each time*)
2	Correct with evidence of multiplicative thinking expressed in words (e.g., *two times the Size number plus two*) or in symbols (e.g., N = 2 x size + 2)

BOARD ROOM TABLES [ABRT5]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but some attempt to use the rule (words or symbols) for the number of tables (e.g., N = 2 x Size + 2 but error in transposing or incomplete written explanation)
2	Correct with evidence of multiplicative thinking expressed in words (e.g., *two less than the number of tables divided by two*) or in symbols (e.g., S = (N – 2)/2)

BOARD ROOM TABLES [ABRT6]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct (No) with little or no reasoning
2	Correct (No) based on specific examples (e.g., tries at least two odd number)

3	Correct (No) with reasoning that recognises Size is half of two less than the number of tables, so it cannot be odd.

BOARD ROOM TABLES [ABRT7]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect or correct (Size 16 or 16) with little or no reasoning
2	Correct response based on extending table pattern, drawing table or by finding the number of tables then finding the Size (i.e., two step solution)
3	Correct response with reasonable explanation either in words (e.g., *divide the number of people by four then take away two*) or in symbols (e.g., S = P/4 – 2)

BOARD ROOM TABLES [ABRT8]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but some evidence of multiplicative thinking (e.g., recognises division is involved but unable to specify correctly, may or may not recognise subtraction)
2	Correct but expressed as two rules in either words (e.g., *find the number of tables by halving the number of people and taking away two then find the Size by taking away two and then halving*) or symbols (e.g., N = P/2 – 2 then S = (N - 2)/2)
3	Correct rule with reasonable explanation either in words (e.g., *you divide the number of people by four then subtract two*) or in symbols (e.g., S = P/4 – 2)

BUTTERFLY HOUSE … [BTH]		
TASK:	**RESPONSE:**	**SCORE**
a.	No response or incorrect	**0**
	Correct (28 wings, 7 bodies, 14 feelers)	**1**
b.	No response or incorrect	**0**
	Correct (4 butterflies)	**1**
c.	No response or incorrect	**0**
	Partially correct with some indication of multiplicative thinking (eg, multiplication algorithm attempted), or correct but evidence of additive thinking (e.g., 98+98+98+98)	**1**
	All correct (392 wings, 98 bodies, 196 feelers) with evidence of multiplicative thinking, eg, algorithm applied correctly or efficient computation strategies such as doubling or renaming (e.g., 400-8 for 4x98)	**2**
d.	No response or incorrect	**0**
	Correct (6 butterflies) but working and/or explanation indicative of additive thinking (e.g., make-all, count all strategy), or incorrect with some indication that the task has been understood in terms of multiplication or division	**1**

	Correct (6 butterflies) with clear explanation in terms of other body parts (e.g., "Can't be 7 because not enough feelers")	**2**

CANTEEN CAPERS… [CC]		
TASK:	**RESPONSE:**	**SCORE**
a.	No response or incorrect with no working and/or explanation	**0**
	Incorrect but recognises that there is more than one option, or correct (6 options) with little/no working and/or explanation to support conclusion	**1**
	Correct (6 different options), working and/or explanation indicates answer arrived at additively (e.g., 6 options listed or drawn fairly randomly)	**2**
	Correct (6 options), working and/or explanation clearly indicates systematic approach and/or recognition of Cartesian Product idea (e.g., "It's 2 x 3 because she has 2 choices of roll and for each one she has 3 choices of drink")	**3**
b.	No response or incorrect with little/no working and/or explanation	**0**
	Correct (Yes), but little/no working or explanation to support conclusion (e.g., drawing or list not systematic, not clear that Cartesian Product idea seen as relevant)	**1**
	Correct (Yes), working and/or explanation clearly supports conclusion (e.g., systematic drawing or list, and/or recognition of 24 options in terms of 2x4x3)	**2**

LEMONADE [ALEM1]

SCORE	**DESCRIPTION**
0	No response or irrelevant response
1	Correct response (9 or 9 cups of sugar)

LEMONADE [ALEM2]

SCORE	**DESCRIPTION**
0	No response or irrelevant response
1	Partially correct response (e.g., ratios 1:4 (sugar:lemons) and 1:3 (water:lemons))
2	Correct response (1/4 and 1/3 or *For every lemon you need one quarter cup of sugar and one third cup of water*), working shows use of ratios and/or fractions.

LEMONADE [ALEM3]

SCORE	**DESCRIPTION**
0	No response or irrelevant response
1	Partially correct response (e.g., ratio 1:4) or correct worded explanation but no rule.
2	Correct response (e.g., sugar = 1/4 n or *For every cup of sugar you need 4 times the amount of lemons, so the amount of sugar needed is one quarter the number of lemons*).

HAT CHANCE [SHAT8]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect, little/no reasoning (e.g., it's just luck)
2	Incorrect (*e.g., name is a boy or girl*) but reasoning that recognises variation in some way (e.g., depends on mix, same chance, could be anything)
3	Correct (name is a girl) with either no explanation or explanation does not reference total (e.g., 16 is bigger than 13)
4	Correct, fraction included in explanation (e.g., 16/29 chance)

SPY SQUAD [GSPSQ7]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect due to errors in calculation but recognises relevance of distance divided by speed in some way
2	Correct (0.425 hours, approximately 0.4 hours or 24-26 minutes). Working should show 3.4/8 which gives the hours or some similar approach (e.g., find time for 3 km and 4 km and split the difference)

SPY SQUAD [GSPSQ8]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect due to errors in calculation but recognises relevance of distance divided by speed in some way
2	Correct (0.3375 hours, approximately 0.33 hours or 20 minutes). Working should show 2.7/8 which gives the hours or some similar approach (e.g., recognise fraction as close to 1 third)

SPY SQUAD [GSPSQ9]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Some working to suggest the relevance of Pythagorus' Theorem and/or previous calculations for taxi time but unclear or incomplete, may not arrive at a conclusion
2	Incorrect (stay in taxi) due to errors in calculation in both the use of Pythagorus to find the hypotenuse **and** in calculating time taken to travel from A to B to C in previous questions, may also miscalculate time to run from A to C
3	Incorrect (stay in taxi) OR correct (faster to get out of the taxi and run) but conclusion based on errors in calculation **either** in the use of Pythagoras to find hypotenuse and time to run from A to C **or** in calculating time taken to travel from A to B to C in previous questions

4	Correct, supported by more accurate reasoning/working that takes into account both the running time (running time = distance/speed ≅ 4.3 km/6 km per hour = 43/60 so approximately 43 minutes) and the car time (approximately 25 + 20 minutes = 45 minutes)

PARK MAP [GMAPA]

SCORE	DESCRIPTION
0	No response or irrelevant response, or 6 or 6.5 (squares)
1	About 1200 (m)
2	**Correct:** About 1300 m or 1.3 kms

PARK MAP [GMAPA1]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Considers squares on the map e.g., about 6 squares or 6.5 squares
2	Uses scale with whole squares only e.g., 1200 m
3	Uses scale with part and whole squares e.g., about 1300 m

PARK MAP [GMAPB]

SCORE	DESCRIPTION
0	No response or irrelevant response, or uses count of squares (e.g., 7)
1	About 1400 m
2	Correct About 1600 m or 1.6 kms

PARK MAP [GMAPB1]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Counts squares e.g., about 7 squares
2	Uses scale but treats diagonals as whole squares e.g., about 1400 m
3	Uses scale appropriately e.g., about 1600 m

PARK MAP [GMAPC]

SCORE	DESCRIPTION
0	No response or irrelevant response, or uses count of squares (e.g., 5 or 6)
1	About 600 m
2	Correct about 700 m

PARK MAP [GMAPC1]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Counts squares (e.g., about 5 or 6 squares)
2	Uses scale but treats diagonals as whole squares (e.g., about 1200m)
3	Uses scale appropriately (e.g., about 1100 m)

PARK MAP [GMAPD]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Counting responses (additive thinking) with no reference to scale (e.g., I would count the squares)
2	Takes account of scale but still largely additive (e.g., I would count the squares and times them by 200)
3	Generalises by using a tool (ruler, paper, string etc) or estimation approach and then applies the scale (e.g., “My little finger was about the width of a square, so I worked out how many times my finger went into the distance and then found this times 200 m to get the distance”).

ASSESSING MULTIPLICATIVE THINKING

STUDENT SCORE SHEET OPTION 4

Student Name:	**Year Level:**

Task	**Item**	**Score**	**Comments**
Board Room Tables	ABRT1		
	ABRT2		
	ABRT3		
	ABRT4		
	ABRT5		
	ABRT6		
	ABRT7		
	ABRT8		
Butterfly House	BTHa		
	BTHb		
	BTHc		
	BTHd		
Canteen Capers	CCa		
	CCb		
Lemonade	ALEM1		
	ALEM2		
	ALEM3		
Hat Chance	SHAT		
Spy Squad	GSPG7		
	GSPQ8		
	GSPQ9		
Park Map	GMAPA		
	GMAPA1		
	GMAPB		
	GMAPB1		
	GMAPC		
	GMAPC1		
	GMAPD		

Total Raw Score	
LAF Zone	

ASSESSING MULTIPLICATIVE THINKING

LAF Raw Score Translator Option 4

The following table is provided to enable teachers to locate students in terms of the **Learning and Assessment Framework for Multiplicative Thinking 2021 (LAF)** on the basis of their performance on the Assessment Tasks for Option 4 (blue font indicates revisions to the original LAF).

To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 20 items altogether).

Total Score	LAF Zone	Level Description
53-57	**8**	Can use appropriate representations, language, and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems. Can express more complex multiplicative relationships in words or symbols in simplest form and work with two variables simultaneously and equivalent expressions. Recognises and uses scale appropriately and can use a generalised solution strategy in a new context. Beginning to recognise the relationships between perimeter, area, and volume.
43-52	**7**	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent, and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically. Able to describe multiplicative relationships as rules in words or symbols but may not express this in simplest form. Can reason algebraically and use symbols to describe what is needed to maintain equivalence in an additive relational context. Can use relationships to calculate simpler volumes and explain thinking in procedural terms.
35-42	**6**	Can work with the Cartesian Product (for each) idea to systematically list or determine the number of options. Can solve a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Able to rename and compare fractions in the halving family and use partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts. Able to describe and justify rules involving multiplicative relationships Beginning to generalise patterns and formalise rules involving multiplication but may miss more complex patterns involving a constant or ratio or scales requiring estimation or use of diagonals.
26-34	**5**	Systematically solves simple proportion and array problems suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions. Able to solve simple, 2-step problems using a recognised rule/relationship but finds this difficult for larger numbers. Able to order numbers involving tens, ones, tenths and hundredths in supportive

		context. Able to determine all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. Beginning to work with decimal numbers and percent but unable to apply efficiently to solve problems. Some evidence that multiplicative thinking being used to support partitioning. Beginning to approach a broader range of multiplicative situations more systematically for instance, able to recognise and apply simple ratios to solve problems involving proportion or scale.
17-25	**4**	Solves more familiar multiplication and division problems involving two-digit numbers. Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations. Tends not to explain their thinking or indicate working. Able to partition given number or quantity into equal parts and describe part formally. Beginning to work with simple proportion (e.g., uses % to describe a sample; can make a start, represent problem, but unable to complete successfully or justify their thinking). Beginning to recognise and use generalisations to solve problems but unable to explain or justify thinking. Use simple scales in straightforward situations and recognises the importance of scale in more complex contexts.
8-16	**3**	Demonstrates intuitive sense of proportion. Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving. May list all options in a simple Cartesian product but cannot explain or justify solutions. Uses abbreviated methods for counting groups (e.g., doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions). Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems. Can maintain equivalence across the equals sign and extend patterns but may not be able to explain or explanation relies on additive thinking. Beginning to recognise the importance of scale.
5-7	**2**	Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting, counts large collections efficiently, systematically keeps track of count, for instance, may order groups in arrays or as a list, but needs to 'see' all groups. Can share collections into equal groups. Recognises small numbers as composite units (e.g., can count equal groups, skip count by twos, threes, and fives). Can extend an additive pattern. Recognises multiplication is relevant but tends not to be able to follow this through to solution. Can list some of the options in simple Cartesian Product and chance situations. Some evidence of MT as equal groups/shares seen as entities that can be counted systematically. Beginning to recognise statistical variation and has some understanding of chance.
0-4	**1**	Can solve simple multiplication and division problems involving relatively small whole numbers, but tends to rely on drawing, models, and count-all strategies. May use skip counting (repeated addition) for groups less than 5. Can make simple observations from data given in a task and extend a simple pattern number pattern. Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.