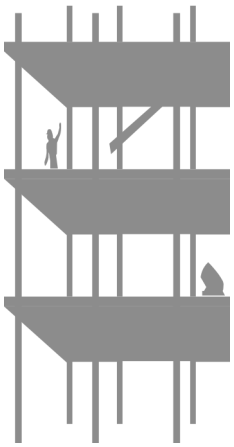


Assessment Materials for Multiplicative Thinking

Assessment Task Booklet
Option 3

GROWING Mathematically

Based on the *Scaffolding Numeracy in the Middle Years* and *Reframing Mathematical Futures II* Research projects

ASSESSING MULTIPLICATIVE THINKING

ASSESSMENT TASK BOOKLET FOR MULTIPLICATIVE THINKING
OPTION 3

NAME
YEAR LEVEL:

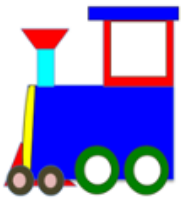
This booklet contains an Extended Task and 5 Supplementary or Short Tasks:

X1 – Trains
S1 – Adventure Camp
S2 – Stained Glass Windows
S3 – Relations
S4 – Skin Rash
S5 – Enlarging Nets

INSTRUCTIONS:

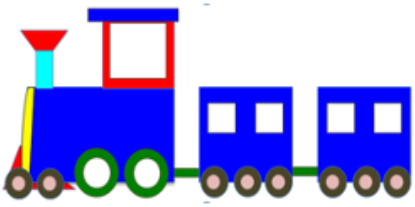
1. Please do as much of each task as you can. Some tasks you will find easy; others will be more difficult.
2. All working must be shown in this booklet. If you need more space, please use the back of the previous page or another space, but make sure we know where to find your answer.
3. When you are asked to **show all your working and explain your answer in as much detail as possible** or to **explain your reasoning using as much mathematics as you can** or **show all your working so we can understand your thinking** do your best to write down what you did and why, in the space provided.
4. Don't rub out any work that you think is incorrect. Simply draw a line through it.
5. If you have any questions please ask your teacher.

Trains



toy train size 1

toy train size 2



toy train size 3

The engine of the train has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each train:

Train size	1	2	3	4	5	6
Number of wheels	8	14				

[ATRNS1]

Fill in the table to show the number of wheels for the trains size 3, 4, 5 and 6.

[ATRNS2]

The largest train set in the toy shop is size 15.

How many wheels does the size 15 have? _____

Show your reasoning.

[ATRNS3]

Ben says his train has exactly 60 wheels. Can Ben be correct? _____

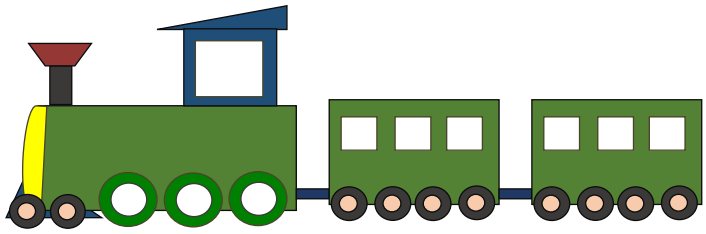
Explain your reasoning.

[ATRNS4]

Write down in words or symbols a rule for working out how many wheels any sized train would need.

[ATRNS5]

The toy shop decides to introduce a new Super Train set. In the new train set the engine has 10 wheels and each carriage has 8 wheels.



Super Train Size 3

How many wheels does a Super Train Size 7 have?

Show your reasoning.

[ATRNS5A]

Write in words or symbols a rule for working out how many wheels any sized Super train would need.

Explain your reasoning.

[ATRNS6]

Write a rule in words or symbols for working out the size of a Super Train given any number of wheels. Explain your reasoning.

Adventure Camp [adc]

Camp Reefton offers 4 activities. Everyone has a go at each activity early in the week. On Thursday afternoon students can choose the activity that they would like to do again.

The table shows how many students chose each activity at the Year 5 camp and how many chose each activity at the Year 7 camp a week later.

	Rock Wall	Canoeing	Archery	Ropes Course
Year 5	15	18	24	18
Year 7	19	21	38	22

Camp Reefton Thursday Activities

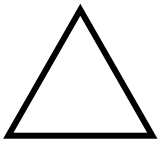
[adca]

a. What can you say about the choices of Year 5 and Year 7 students?

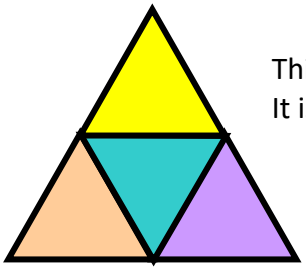
[adcb]

The Camp Director said that canoeing was more popular with the Year 5 students than the Year 7 students. Do you agree with the Director's statement? **Use as much mathematics as you can to support your answer.**

Stained Glass Windows ...



Stained glass windows can be made using small triangles.



This stained glass window is made from four small triangles joined together. It is 2 triangles wide at the base and 2 triangles high.

[sgwa]

How many small triangles will you need if your window is to be 4 triangles wide and 4 triangles high?

[sgwb]

Part of the stained glass window shown below, is hidden by a sign. How many small triangles were needed to make this window?

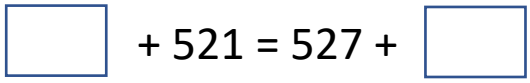
[sgwc]

How would you advise a friend on how to work out the number of small triangles that would be needed for a window 26 triangles wide?

Relations

[ARELS1]

What numbers would go in these boxes to make a true number sentence (the numbers may be different). Explain your reasoning.



[ARELS2]

Find a different pair of numbers that would make the number sentence above true.

[ARELS3]

Describe how you could find all possible pairs of numbers that would make this a true number sentence.

Skin Rash

[SRASH]

A bottle of medicine has printed on it:

> ***WARNING: For applications to skin areas there is a 15% chance of getting a rash. If you get a rash, consult your doctor.***

What does this mean?

(A) Don't use the medicine on your skin - there's a good chance of getting a rash.

(B) For application to the skin, apply only 15% of the recommended dose.

(C) If you get a rash, it will probably involve only 15% of the skin.

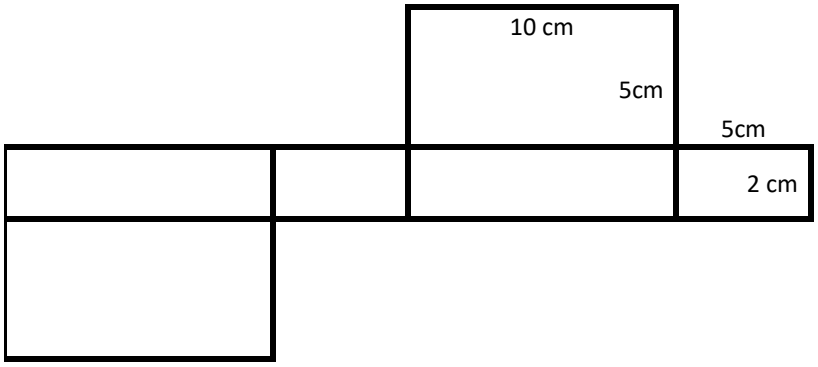
(D) About 15 out of every 100 people who use this medicine get a rash.

(E) There is hardly any chance of getting a rash using this medicine.

Explain your reasoning

Enlarging Nets

This is the net of a rectangular prism which would fold up to make a box. The units are shown in cm.



[GENLG0]
Draw a net of a rectangular prism that would fold to make a box twice as high.

[GENLGa]
What is the length of the **longest** side of your new net?

[GENLGb]
What is the area of the smallest face of your new net? Show all your working.

[GENLGc]
Find the volume of your new net. Show all your working.

[GENLGd]
Explain how you could make a box that is twice as big as the one shown.

ASSESSING MULTIPLICATIVE THINKING

SCORING RUBRIC OPTION 3

TRAINS [ATRNS1]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	At least two entries correct
2	Table completed correctly (20,26,32,38)

TRAINS [ATRNS2]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (92) with no explanation/working or incorrect response with working to show some understanding of pattern or incorrect with working to show minor calculation error
2	Correct response with an explanation that reflects the use of an additive strategy (e.g., *goes up by 6* or continues table to a train size of 15)
3	Correct response with an explanation of a multiplicative approach expressed in words or as a rule but not in simplest form (e.g., *you multiply 6 by 14 and add 8 or working to show 6 x 14 + 8*)
4	Correct response with an explanation of a multiplicative approach expressed in words or as a rule in simplest form that recognises the 6 wheels in the engine (e.g., *you need to times 15 by 6 and add 2* or 15 x 6 + 2)

TRAINS [ATRNS3]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (No) but with no explanation OR incorrect response with sound reasoning
2	Correct response with reasoning to support conclusion (e.g., *60 – 8 is 52 and 52 is not divisible by 6* or *a size 9 train would have 56 wheels and a size 10 train would have 62 wheels so you can't have a train with 60 wheels*.

TRAINS [ATRNS4]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	General statement (e.g., *it goes up by 6*) OR incorrect but some evidence that multiplication involved, may or may not recognise addition

2	Rule correctly expressed in words (e.g., *you multiply 6 by one less than the train Size number and you add 8*) or in symbols but not in simplest form (e.g., N = 8 + 6S – 6 or N = 8 + 6 (S – 1))
3	Rule correctly expressed in words or symbols in simplest form that recognises the 6 wheels in the engine (e.g., *you need to times S by 6 and add 2 or N = 6S + 2*)

TRAINS [ATRNS5]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (58) with little/no explanation/working OR incorrect response with working to show some understanding of pattern, may involve a minor calculation error
2	Correct response with an explanation that suggests the use of an additive strategy (e.g., *goes up by 8* or uses a table for Super Train Sizes from 1 to 7)
3	Correct response with an explanation that indicates a multiplicative approach expressed either in words (e.g., *you multiply 8 by one less than the Size and you add 10)* OR symbols (e.g., 10 + 6 x 8 or 2 + 7 x 8)

TRAINS [ATRNS5A]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	General statement (e.g., *it goes up by 8*) OR incorrect but some evidence that multiplication involved, may or may not recognise addition
2	Rule correctly expressed in words (e.g., *you multiply 8 by one less than the Super train Size number and you add 10*) or in symbols but not in simplest form (e.g., N = 10 + 8S – 8 or N = 10 + 8 (S – 1))
3	Rule correctly expressed in words or symbols in simplest form that recognises the 8 wheels in the engine (e.g., *you need to times S by 8 and add 2 or N = 8S + 2*)

TRAINS [ATRNS6]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but some evidence of multiplicative thinking (e.g., recognises division is involved but unable to specify correctly, may or may not recognise subtraction)
2	Correct rule with reasonable explanation either in words (e.g., you take 2 from the number of wheels and divide by 8) or in symbols (e.g., S = (N – 2)/8)

ADVENTURE CAMP … [adc]		
TASK:	**RESPONSE:**	**SCORE**
a.	No response or incorrect or irrelevant statement	**0**

	One or two relatively simple observations based on numbers alone (e.g., "Archery was the most popular activity for both Year 5 and Year 7 students", "More Year 7 students liked the rock wall than Year 5 students")	**1**
	At least one observation which recognises the difference in total numbers (e.g., "Although more Year 7s actually chose the ropes course than Year 5, there were less Year 5 students, so it is hard to say")	**2**
b.	No response	**0**
	Incorrect (No), argument based on numbers alone (e.g., "There were 21 Year 7s and only 18 Year 5s")	**1**
	Correct (Yes), but little/no working or explanation to support conclusion	**2**
	Correct (Yes), working and/or explanation indicates that numbers need to be considered in relation to respective totals (e.g., "18 out of 75 is more than 21 out of 100"), but no formal use of fractions or percent or further argument to justify conclusion	**3**
	Correct (Yes), working and/or explanation uses comparable fractions or percents to justify conclusion (e.g., "For Year 7 it is 21%. For Year 5s, it is 24% because 18/75 = 6/25 = 24/100 = 24%")	**4**

STAINED GLASS WINDOWS … [sgw]		
TASK:	**RESPONSE:**	**SCORE**
a.	No response or incorrect with no working and/or explanation	**0**
	Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation	**1**
	Correct (16 triangles), with evidence of additive reasoning based on drawing and counting	**2**
	Correct (16 triangles), with evidence of multiplicative reasoning based on 4 by 4	**3**
b.	No response or incorrect with little/no working and/or explanation	**0**
	Incorrect based on inaccurate drawing and/or counting of triangles, or correct with little/no explanation	**1**
	Correct (81 triangles), with evidence of additive reasoning based on drawing and counting, or inappropriate use of area formula (e.g., L x W)	**2**
	Correct (81 triangles), with evidence of multiplicative reasoning based on pattern (e.g., 9 by 9)	**3**
c.	No response or incorrect	**0**
	Advice based on additive thinking (e.g., "1 less each time you go up")	**1**
	Correct (advice based on rule (e.g., 26 X 26)	**2**

RELATIONS [ARELS1]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect response but suggest the difference of 6 is recognised in some way (e.g., *add 6 to the right-hand side*)
2	Two correct numbers given (e.g., 13 and 7; 527 and 521) but little/no reasoning.
3	Two correct numbers given where the number on the left is 6 more than the number on the right (e.g., 100 and 94) with reasoning that reflects the relationship between 521 and 527 (difference of 6).

RELATIONS [ARELS2]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	A different and correct pair.

RELATIONS [ARELS3]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incomplete attempt based on previous answers (e.g., *add 2 more to both*).
2	Statement regarding the difference of 6 (e.g., *number on the left must be six more than the number on the right)* or expression showing the difference (e.g., a + 6, and a)

SKIN RASH [SRASH]

SCORE	DESCRIPTION
0	No response or clearly incorrect, that is, B OR C or multiple selections excluding A, D and E
1	Indicates some understanding of chance expressed as a percentage, that is, either E or A , supported by reasonable explanation (e.g., *E because 15 out of 100 is quite small*, or *A because you don't want to get a skin rash and 15% is quite a high chance*)
2	Correct (D) supported by a reasoning that refers to sample in some way (e.g., *15% of all the people who used this got a rash*). May choose D & A or D & E

ENLARGING NETS [GENLG0]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Draws a net that would fold to make a box with one side length doubled (could be any side)

ENLARGING NETS [GENLGa]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	20cm or 10cm depending on the side chosen to double.

ENLARGING NETS [GENLGb]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Focus on a face of original prism (e.g., 50 or 10) or adds lengths together (e.g., 22), typically without showing units. May not show working.
2	Multiplies correctly but chooses incorrect face (e.g., 40 cm^2 [10 x 4]; 100 cm^2 [10 x 10]). May not show units.
3	Correct (20cm^2). All working and units shown.

ENLARGING NETS [GENLGc]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Any kind of additive thinking using either original prism or new prism (e.g., 10 + 5 + 2)
2	Finds volume of the original prism (10 x 5 x 2 = 100 cm^3). May not show all units.
3	Correct (200 cm^3). All working and units shown.

ENLARGING NETS [GENLGd]

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Repeats the instruction (e.g., Make it double the size) or focusses on length (e.g., Double the length)
2	Explanation recognises the need to double all side lengths or may compare new volume (200 cm^3) to original volume (100 cm^3).
3	Explanation recognises the need to double all side lengths and acknowledges that area and/or volume will be more than double. May go further and question what is meant by “double”.

ASSESSING MULTIPLICATIVE THINKING

STUDENT SCORE SHEET OPTION 3

Student Name:	**Year Level:**

Task	Item	Score	Comments
Trains	ATRNS1		
	ATRNS2		
	ATRNS3		
	ATRNS4		
	ATRNS5		
	ATRNS5A		
Adventure Camp	adca		
	adcb		
Stained Glass Windows	swga		
	swgb		
	swgc		
Relations	AREL1		
	AREL2		
	AREL3		
Skin Rash	SRASH		
Enlarging Nets	GENLG0		
	GENGLa		
	GENLGb		
	GENLGc		
	GENLGd		

Total Raw Score		
LAF Zone		

Notes:

ASSESSING MULTIPLICATIVE THINKING

LAF Raw Score Translator Option 3

The following table is provided to enable teachers to locate students in terms of the **Learning and Assessment Framework for Multiplicative Thinking 2021 (LAF)** on the basis of their performance on the Assessment Tasks for Option 3 (blue font indicates revisions to the original LAF).

To use the table you will need to determine each student's total score by adding the rubric scores assigned to each item (there are 20 items altogether).

Total Score	LAF Zone	Level Description
47-52	**8**	Can use appropriate representations, language, and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. Can justify partitioning. Can use and formally describe patterns in terms of general rules. Beginning to work more systematically with complex, open-ended problems. Can express more complex multiplicative relationships in words or symbols in simplest form and work with two variables simultaneously and equivalent expressions. Recognises and uses scale appropriately and can use a generalised solution strategy in a new context. Beginning to recognise the relationships between perimeter, area, and volume.
35-46	**7**	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. Can solve and explain solutions to problems involving simple patterns, percent, and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems. Locates fractions using efficient partitioning strategies. Beginning to make connections between problems and solution strategies and how to communicate this mathematically. Able to describe multiplicative relationships as rules in words or symbols but may not express this in simplest form. Can reason algebraically and use symbols to describe what is needed to maintain equivalence in an additive relational context. Can use relationships to calculate simpler volumes and explain thinking in procedural terms.
27-34	**6**	Can work with the Cartesian Product (for each) idea to systematically list or determine the number of options. Can solve a broader range of multiplication and division problems involving 2-digit numbers, patterns and/or proportion but may not be able to explain or justify solution strategy. Able to rename and compare fractions in the halving family and use partitioning strategies to locate simple fractions. Developing sense of proportion, but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts. Able to describe and justify rules involving multiplicative relationships Beginning to generalise patterns and formalise rules involving multiplication but may miss more complex patterns involving a constant or ratio or scales requiring estimation or use of diagonals.
23-26	**5**	Systematically solves simple proportion and array problems suggesting multiplicative thinking. May use additive thinking to solve simple proportion problems involving fractions. Able to solve simple, 2-step problems using a recognised rule/relationship but finds this difficult for larger numbers. Able to order numbers involving tens, ones, tenths and hundredths in supportive

		context. Able to determine all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. Beginning to work with decimal numbers and percent but unable to apply efficiently to solve problems. Some evidence that multiplicative thinking being used to support partitioning. Beginning to approach a broader range of multiplicative situations more systematically for instance, able to recognise and apply simple ratios to solve problems involving proportion or scale.
14-22	**4**	Solves more familiar multiplication and division problems involving two-digit numbers. Tend to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations. Tends not to explain their thinking or indicate working. Able to partition given number or quantity into equal parts and describe part formally. Beginning to work with simple proportion (e.g., uses % to describe a sample; can make a start, represent problem, but unable to complete successfully or justify their thinking). Beginning to recognise and use generalisations to solve problems but unable to explain or justify thinking. Use simple scales in straightforward situations and recognises the importance of scale in more complex contexts.
9-13	**3**	Demonstrates intuitive sense of proportion. Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving. May list all options in a simple Cartesian product but cannot explain or justify solutions. Uses abbreviated methods for counting groups (e.g., doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions). Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems. Can maintain equivalence across the equals sign and extend patterns but may not be able to explain or explanation relies on additive thinking. Beginning to recognise the importance of scale.
4-8	**2**	Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting, counts large collections efficiently, systematically keeps track of count, for instance, may order groups in arrays or as a list, but needs to 'see' all groups. Can share collections into equal groups. Recognises small numbers as composite units (e.g., can count equal groups, skip count by twos, threes, and fives). Can extend an additive pattern. Recognises multiplication is relevant but tends not to be able to follow this through to solution. Can list some of the options in simple Cartesian Product and chance situations. Some evidence of MT as equal groups/shares seen as entities that can be counted systematically. Beginning to recognise statistical variation and has some understanding of chance.
0-3	**1**	Can solve simple multiplication and division problems involving relatively small whole numbers, but tends to rely on drawing, models, and count-all strategies. May use skip counting (repeated addition) for groups less than 5. Can make simple observations from data given in a task and extend a simple pattern number pattern. Multiplicative thinking (MT) not really apparent as no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.