Reframing Mathematical Futures II

Statistical Reasoning Learning Progression - Teaching Advice

Variation underpins all of statistical reasoning. It is the key overarching idea. For the purposes of the Statistical Reasoning Learning Progression and associated teaching advice, this every large idea has been described in ways that link more easily to the curriculum. The three Big Ideas of variation are briefly described here.

- *Variation with Expectation and Randomness, e.g., chance, probability, averages [VER]*

When a dice is thrown we expect variation. We would be surprised if in six throws, the number 2 came up every time. Rolling a dice, spinning a spinner, drawing names or blocks from a bag are random events and we expect the outcomes to be different every time. When we think about averages, however, we tend not to see these in terms of variation and expectation. An average, whether mean, median, or mode, expresses an expectation about a data set. The average is a description that typifies the data. It varies as new data are added or data points are removed.

- *Variation with Distribution and Expectation, e.g., graphs, tables, representations [VDE]*

Distribution describes the ways in which the data are spread out or distributed. A graph is a picture of a distribution and the variation it represents. There are many ways of representing data distributions and some of these are highly technical and used only by professional statisticians. In school it is useful to develop the idea that tables, graphs or other representations are ways of visualising variation through the distribution of the data.

- *Variation with Informal Inference, e.g., sampling, populations, decision-making [VII]*

Collecting data is a purposeful activity that describes a situation and can help people make decisions. When considering any data set we need to consider aspects such as the sample, how the data were collected, and the questions we are asking about the data. Professional statisticans use a variety of tools to draw inferences from data but at the school level the aim is to encourage students to make informal inferences with justifications based on the data, or to ask questions about the nature of the data.

This teaching advice document is organised around these three focus areas. Teaching activities address more than one of these areas and it may depend on the class and prior understandings as to which focus you need to take.

In general, students at all levels should be provided with opportunities to undertake complete data investigations using the PPDAC cycle of **P**ose a question, **P**lan the investigation to answer that question, collect the **D**ata, **A**nalyse the data, draw a **C**onclusion and write a report.

ZONE 1 Description	Teaching Implications
VER *Is familiar with standard simple probabilistic/chance situations (e.g., dice, coins) but uses these at a superficial level (e.g., the chance of getting HT when tossing two coins is the same as HH or TT).* ***VDE*** *Reads a single value from a simple graph on either the x or y axis (e.g., SHWKA1.1, SHGT1.1) but does not look at axes (x, y) values simultaneously. Tends to focus on the highest values unless directed.* *Variation is considered visually (e.g., in SGHT2 is likely to choose School B as having more variation) but not otherwise recognised.* ***VII*** *Can give real world examples of variation (e.g., weather).*	**Consolidate and Establish** ➢ **Address variation explicitly** in a variety of contexts and representations, e.g., have the whole class measure the armspan of a single student, record results and discuss the differences. Conduct dice and coin tossing experiments, record values and compare results across the class. ➢ *Licorice Lines* activity *[https://www.resolve.edu.au/statistics-licorice-lines].* ➢ **Explore predictions**, e.g., What might happen if another student's armspan was measured? Would weather patterns be the same if they were collected in another town? Measure the height of every student in the class. How would the variation in the data compare with the measurements on the individual student? Predict how similar this would be if we did this in a different class? How could we show this to another person? **Introduce and Develop** ➢ **Present data in a variety of ways** and move between these, e.g., collect data from tossing a single die many times; represent this data in different ways such as tally marks, table, graph, etc. Discuss differences among students' representations. ➢ Toss two coins together and represent possible outcomes (HH, TT, HT) from ten tosses as a graph. Combine data with other students' data and redraw the graph. Use **tree diagrams** to show what the probability is and discuss the students' representations in the light of the tree diagram, stressing uncertainty. ➢ **Undertake a range of chance experiments** using different kinds of random generator (dice, coins, spinner, etc.) and record the outcomes in different ways. ➢ **Tell stories from simple graphs**, paying attention to details, e.g., *Tell Me a Story* (MCTP)[1] ➢ *You tell the story* (nrich) http://nrich.maths.org/4802 ➢ **Create graphs that tell given stories** e.g., We measured the calf every day. At first its weight gained very rapidly but after a week the gain slowed down. ➢ *Temperature Graphs* (Maths300). ➢ *Licorice Lines* activity [https://www.resolve.edu.au/statistics-licorice-lines]. ➢ **Explore the language of uncertainty** and what is meant by luck, e.g., discuss what it means to be lucky and how you could decide what is the chance of an event happening. What does 70% chance of rain mean? What is a 1-in-100 years flood? Does it mean that

[1] http://www.canberramaths.org.au/uploads/1/1/7/0/11707964/tell_me_a_story_post-primaryvolume_1_pp253-256.pdf

	you will have to wait 100 years for the same flood level to occur? Order chance phases on a number line from 0 to 1. ➢ Undertake a range of chance experiments using different kinds of random generator (dice, coins, spinner, etc.) and discuss the likelihood of a particular outcome, stressing the quantitative aspects. What does it mean for events to be equally likely? Go beyond two outcomes (50:50) and include opportunities for different types of equally likely outcomes (e.g., 1/3, 1/3, 1/3) using a die or spinners. Represent the results in different ways (graph, tree diagram, etc.) ➢ Undertake *What's in the Bag* investigation *[Maths300] or Mystery Bag* [AAMT TopDrawer]. Focus on the sample size.

ZONE 2 Description	Teaching Implications
VER *Uses reasoning that recognises variation in some way but may not appreciate expectation (chance) (e.g., SCON1A.1, SCON1B.1).* *Is familiar with simple chance experiments (e.g., SD12B) but is unable to reason quantitatively and is likely to rely on personal beliefs rather than the data when explaining an outcome, e.g., it's the way you roll the die (SD12B).* *Uses the language of 50% or 50:50 but does not appreciate the meaning (SBOX9.1).* ***VDE*** *Reads information from simple graphs using x and y axes, and describes what the graph is about (It's about the time spent on homework) but may not recognise association between variables (e.g., SHWKB.1).* ***VII*** *Recognises one aspect of sampling (e.g., either size or method) in a familiar context (SMV10.1) but does not coordinate these ideas to provide a random and representative sample.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Develop ideas about sampling**, e.g., Discuss examples and non-examples of appropriate sample size and method; e.g., Should we survey everyone in the school about the uniform? How many students should we survey? How should we choose them? Focus on developing ideas about random and representative samples. ➢ **Focus on fairness and randomness**, e.g., Have a class debate about what it means to be fair in non-statistical and statistical contexts; e.g., Dad may get a bigger portion of dinner – is that fair? ➢ **Discuss questions of fairness**, e.g., *Dicey Differences [https://nzmaths.co.nz/resource/dicey-differences]*. How can we determine that a die is fair? ➢ *Dice Duels* [https://topdrawer.aamt.edu.au/Statistics/Activities/Dice-duels] ➢ Play *Rock, Paper, Scissors* [https://www.resolve.edu.au/probability-rock-paper-scissors] and discuss the fairness of the game. ➢ **Research and discuss random events**, e.g., if you are collecting particular stickers from a supermarket is there an equal chance of getting each of them? Are they distributed randomly? ➢ **Undertake simple repeat sampling experiments** such as *What's in the Bag [Maths 300]*. When is this unfair? ➢ **Explore ideas about distribution**, e.g., colours in M & M packets. Record the findings and discuss the results in terms of randomness. *Balancing Act* investigation [https://topdrawer.aamt.edu.au/Statistics/Good-teaching/Making-informal-inferences/Single-measurement-variables/B]. ➢ **Coordinate expectation and variation**, e.g., conduct large scale experiments of dice rolling or coin tossing and show how the variation approaches expectation in large trials (use computer simulation or combine students' individual experiments); *What do you know about probability? (2)* (nrich) https://nrich.maths.org/12151 ➢ **Make predictions** based on expectation and variation, e.g., *What's in the Bag?* [Maths 300]

ZONE 3 Description	**Teaching Implications**
VER *Recognises expectation but for chance interprets this in terms of strict probability (e.g., SD12A.2) and for statistical situations knows the value of mean/median (e.g., SAMED.1, SAMEA.1) but explanations are limited or based on a visual representation showing the range (SHGT2.2).* *Applies ideas of variation drawing on expectation but in a limited way (e.g., RUTH.1; SCON1B.2; focuses on sum to 60 rather than realistic numbers in SD12A.2; not recognising with proportion SHAT8.3; general statements SCLIM1.1).* ***VDE*** *Recognises variation but in graphing situations may explain this based on visual representation (SHGT3.1).* *Reasons quantitatively using direct comparison but relies on additive thinking (e.g., not recognising scale in STWN1.3).* ***VII*** *In more complex inference tasks or less familiar contexts draws on opinion rather than data (SSKIN.1; STWN1.1) or retreats to "luck" as an explanation (STATS.1).*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Pose questions that can be answered by collecting data** through providing opportunities for quantitative reasoning in statistical and probabilistic contexts e.g., *Are Males better drivers?* [https://topdrawer.aamt.edu.au/Statistics/Activities/Are-males-better-drivers]; telling stories from graphs, *How long is a piece of string?* [https://topdrawer.aamt.edu.au/Statistics/Activities/How-long-is-a-piece-of-string]. How many hours does the typical Year X student sleep? What are student opinions on an issue important in the school? ➢ **Collect and analyse statistical data relevant to the class** e.g., *Licorice Lines* [[https://www.resolve.edu.au/statistics-licorice-lines]; *Balancing Act* [https://topdrawer.aamt.edu.au/Statistics/Good-teaching/Making-informal-inferences/Single-measurement-variables/B]; collect Handspan and Foot length data and discuss relationship. Represent data in different ways (lists, tables, graphs). Compare attributes, e.g., right hand vs left hand, boys vs girls. ➢ **Compare groups** through comparing whole distributions , e.g., *Balancing Act* Comparing Groups [https://topdrawer.aamt.edu.au/Statistics/Activities/Balancing-act-2]; *Licorice Lines* activity [https://www.resolve.edu.au/statistics-licorice-lines]; *The Hair Colour Game* (nrich) https://nrich.maths.org/6964 ➢ **Extend the language of uncertainty**, e.g., Read *Pigs Might Fly* by Emily Rodda, discuss the language, ask students to write a short story using words related to chance and uncertainty. Research the meaning and origins of common chance phrases such as 'Once in a Blue Moon', 'Pigs Might Fly', etc. Discuss which sayings have a quantitative basis and which are just about luck. ➢ *Friday 13th* (nrich) https://nrich.maths.org/610 ➢ **Develop the relationship between uncertainty and statistical information** e.g., The *Birth Month Paradox* [Maths300]; *Flipping Coins* [https://www.abc.net.au/science/surfingscientist/pdf/lesson_plan19.pdf]

ZONE 4 Description	Teaching Implications
VER *Recognises variability and expectation in more complex random situations (e.g., SD12B.2) but explanation refers to uncertainty in general terms and is not quantified or is based on strict probability (expectation) (e.g., SD12B.3). May not recognise the importance of equal likelihood (e.g., STATS.2). Recognises relative order in language of uncertainty (WORD.1) but does not appreciate some subtleties. Reasons quantitatively in familiar situations involving related comparison and in the context of uncertainty (e.g., SBOX9). Relies on additive thinking in situations involving measures of central tendency (SAMEA.2), and is unlikely to question the quality of data (SAOUT1). Falls back on personal beliefs in more complex situations when asked for an explanation (e.g., SCON3.1).* ***VDE*** *Compares data in two graphs but focuses on single elements only (STWN2.1). Can associate two variables with single value (SHWKB.2) and provides descriptive explanations (SHWKC.1).* ***VII*** *Critiques sampling approaches using single aspects only (i.e., size or method) in an evaluative situation (e.g., SMV11.1; SMV12.1; SMV13.1).*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Make hypotheses** that can be tested by collecting data, e.g., e.g., *Are Males better drivers?* [https://topdrawer.aamt.edu.au/Statistics/Activities/Are-males-better-drivers]; *Leisure Survey* [http://www.scootle.edu.au/ec/viewing/L3154/index.html] ➢ **Explore fairness** through different sampling methods e.g., *Winning the Lottery* (nrich) http://nrich.maths.org/7244; compare outcomes from dice and spinners; fair games, e.g., *Odds or Sixes* (nrich) http://nrich.maths.org/2859 ➢ **Explore the role of context** with a focus on language, questioning, and the meaning of numbers relative to context (e.g., The average family has 2.3 children? What does this mean?) ➢ **Construct a variety of graphs** from stories, news articles, e.g., use http://graphingstories.com/ to provide videos that students can then graph. Write a short report about the process. ➢ **Explore large samples with technology** to make predictions about increasingly complex situations, e.g., data from the Melbourne Cup or the Bureau of Meteorology. ➢ **Describe "what's typical"** e.g., through the idea of the 'middle 50%" in Balancing Act [https://topdrawer.aamt.edu.au/Statistics/Activities/Balancing-act-2] taking account of the context; *Consensus* [https://education.abc.net.au/home#!/media/687034/consensus]

ZONE 5 Description	**Teaching Implications**
VER *Provides a statistical explanation but this may be incomplete (e.g., SAMEA.3), and recognises equal outcomes for all numbers (SCON3.2).* *Recognises simple proportion in chance contexts (BOX9.3).* *Orders language of chance qualitatively (WORD.2).* *Recognises key aspects of central tendency but reverts to non-statistical justifications (SAMED.2, SAOUT.2).* *Implicitly recognises that all combinations of numbers have the same chance (STATS3).* ***VDE*** *Intuitively suggests association expressed in non-quantitative way (e.g., SSKIN.2), and can recognise important information in making comparisons (STWN3.2).* *Recognises relevant aspects of graphical representation and uses these to reason statistically (e.g., spread in STWN2.2 or STWN1.2) but may not include all aspects (SGHT3.3).* ***VII*** *Recognises appropriate sample size (SCLIM1.2) and provides appropriate critiques of sampling method (SMV10.2, SMV12.2) but does not explicitly include randomness.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Critique different graphical representations** to interpret, describe, and compare affordances and constraints of different representations, e.g *What's going on in this graph?* [https://www.nytimes.com/column/whats-going-on-in-this-graph]; *Misleading Graphs* [https://topdrawer.aamt.edu.au/Statistics/Misunderstandings/Misleading-graphs]. *Leisure Survey* [http://www.scootle.edu.au/ec/viewing/L3154/index.html]. ➢ Introduce the **relationship between statistical data and algebra** through function graphs, e.g., *Guessing the graph* (nrich) https://nrich.maths.org/6990; the relationship between handspan and foot length; distance-time graphs. ➢ **Represent and quantify relationships between two variables**, e.g., use percentages in two way tables; Collect data about 'favourites' such as icecream flavours, food, TV shows, etc., by boys, girls or Year 7/Year 8. Start with only two choices to create 2x2 tables. Then expand to more choices. ➢ **Critique relationships in non-graphical forms** e.g., 2-way tables, nested tables; Explore the ABS census website and critique different representations of data (http://www.abs.gov.au); *Consensus* [http://concensus.splash.abc.net.au/concensus-game/]. ➢ **Examine and explore different distributions** to make variability explicit, e.g., *Digging into Australian Data with TinkerPlots* datasets. Explore probability distributions using simple experiments or simulations (e.g., racetrack, sum of two dice) to show non-uniform distributions. ➢ **Draw pictures of distributions** that recognise unevenness/variations in distributions e.g., sketch what you think the distribution of handspans would look like. Collect data and compare with sketches.

ZONE 6 Description	Teaching Implications
VER *Constructs reasonable arguments based on an understanding of chance and probability (SCON3.3) and context (SBED.2).* *Uses measures of central tendency to justify a closed response (AMED.3; SHSE3.1).* ***VDE*** *Interprets and describes the association between two variables and considers the implication (e.g., SHWKB.3; SM8GR.2) in visual contexts. Beginning to work with the association between two variables in non-graphical format (SSKIN.3).* ***VII*** *Provides sensible critique of sampling in context of method and sample size but is implicit rather than explicit about randomness (SMVE11.2).*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ Introduce the differences between **observed and expected values** e.g., compare the distribution of heads from 10 coin flips with the theoretical value. ➢ **Simulate real situations** using random generators, e.g., *Birth Month Paradox* [Maths300]; Coke vs Pepsi taste test https://serc.carleton.edu/sp/cause/datasim/examples/cokepepsi.html. Simulate the test using coin tosses. ➢ **Focus on proportion**, e.g., using % when comparing attributes; visit the ABS Census site and use the data provided to move from % to counts and write a news story about a particular place. ➢ **Describe and make predictions about distributions**, e.g., Using the mean, median and mode predict what a data set might look like as a stacked dot plot – *Balancing Act* could provide suitable data. Spinners can be used to create non-uniform distributions. ➢ *Which List Is Which?* (nrich) https://nrich.maths.org/7731 ➢ Further develop the ideas of **observed and expected values**, e.g., Consider 'favourites' data and compare observed values with predicted values if there were no difference between the categories. ➢ **Create hypotheses** from expected values or real situations and explore quantification strategies to test these, e.g., seeds with whirligig dispersion mechanisms will disperse more widely than those with a single 'sail'. Many science experiments lend themselves to this type of activity. *Birth Month Paradox* [Maths300]. ➢ *Louis' Ice Cream Business* (nrich) https://nrich.maths.org/9599

ZONE 7 Description	Teaching Implications
VER *Recognises variation appropriately including ideas of fairness, equality of outcomes etc. as appropriate (SCON2.3; SRUTH.4; STATS.4)* *Uses all available data to justify decisions or evaluations statistically (e.g., SSKIN.4; SAMEA.4; SAOUT.3).* *Uses data to justify responses and recognises limitations (SAMEA.4; SAOUT.3) but may still revert to offering an opinion based on individual beliefs (e.g., STATS.4).* *Recognises variability relative to context and the nature of a distribution to provide a realistic solution (SCON2.3).* ***VDE*** *Recognises and describes the spread of data explicitly in a statistical sense using statistically important information not just a visual image (STWN2.3; STWN3.3).* ***VII*** *Makes reasonable sampling decisions that recognise the importance of randomness (e.g., SMV10.3), and critiques inappropriate non-representative and/or non-random samples.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Quantify conjectures** with a focus on proportional reasoning, e.g., What proportion of students walks to school? Would it be 25%, 35%? More than that? How could we find out? *Capture-and Recapture* (nrich) https://nrich.maths.org/9609. ➢ **Ask questions about uncertainty** in statistical findings *e.g., How Confident are you?* (nrich) https://nrich.maths.org/9685 ➢ **Work with more complex ways of displaying data**, e.g., nested tables, dual graphs such as temperature/chance of rain graphs, box plots. Look at *Designing with data* [https://ft-interactive.github.io/visual-vocabulary/]. Discuss the clarity and effectiveness of different data displays. Focus on proportionality of box and whisker plots. Census at School data may be useful (from either the UK, www.censusatschool.com, or New Zealand, https://new.censusatschool.org.nz) ➢ **Relate expected values to theoretical outcomes** e.g., *Birth Month Paradox* [Maths300]; *Capture and recapture* (nrich) https://nrich.maths.org/9609. ➢ **Use complex proportional reasoning** that involves variation in real data beyond the classroom context (Census at School, layer datasets, Australian data from ABS)

ZONE 8 Description	Teaching Implications
VER *Recognises variability relative to chance context using proportional reasoning to support arguments (SD12B.4, SCON3.4. SCON1B.3)* *Recognises, coordinates and integrates all relevant information to make evidence based decisions using relevant context (SBED.3).* ***VDE*** *Recognises, coordinates and integrates all relevant information to make evidence based decisions using proportional reasoning (SSKIN.4).* *Applies ideas about central tendency to justify explanations and decisions (SHSE1.2, SHSE2.2, SHSE3.2).* *Recognises equal chance and interprets chance situations mathematically rather than offering an opinion (STATS.5).* ***VII*** *Makes appropriate statistical critiques of sampling using size, method, range, representativeness in justification (SMV11.3, SMV13.3).*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Consider more complex social issues**, e.g., Research voting systems (first past the post, proportional representation, Hare-Clark); visit the Bureau of Meteorology site (www.bom.gov.au) where there are archived datasets available (e.g., monthly rainfall for selected areas). ➢ **Introduce resampling processes** for decision making, for example comparing two data sets (memorising meaningful and nonsense words).