Reframing Mathematical Futures II

Statistical Reasoning Learning Progression

Variation underpins all of statistical reasoning. It is the key overarching idea. For the purposes of the Statistical Reasoning Learning Progression and associated teaching advice, this very large idea has been described in ways that link more easily to the curriculum. The three Big Ideas of variation are briefly described here.

- *Variation with Expectation and Randomness, e.g., chance, probability, averages [VER]*

When a dice is thrown we expect variation. We would be surprised if in six throws, the number 2 came up every time. Rolling a dice, spinning a spinner, drawing names or blocks from a bag are random events and we expect the outcomes to be different every time. When we think about averages, however, we tend not to see these in terms of variation and expectation. An average, whether mean, median, or mode, expresses an expectation about a data set. The average is a description that typifies the data. It varies as new data are added or data points are removed.

- *Variation with Distribution and Expectation, e.g., graphs, tables, representations [VDE]*

Distribution describes the ways in which the data are spread out or distributed. A graph is a picture of a distribution and the variation it represents. There are many ways of representing data distributions and some of these are highly technical and used only by professional statisticians. In school it is useful to develop the idea that tables, graphs or other representations are ways of visualising variation through the distribution of the data.

- *Variation with Informal Inference, e.g., sampling, populations, decision-making [VII]*

Collecting data is a purposeful activity that describes a situation and can help people make decisions. When considering any data set we need to consider aspects such as the sample, how the data were collected, and the questions we are asking about the data. Professional statisticans use a variety of tools to draw inferences from data but at the school level the aim is to encourage students to make informal inferences with justifications based on the data, or to ask questions about the nature of the data.

This teaching advice document is organised around these three focus areas. Teaching activities address more than one of these areas and it may depend on the class and prior understandings as to which focus you need to take.

In general, students at all levels should be provided with opportunities to undertake complete data investigations using the PPDAC cycle of **P**ose a question, **P**lan the investigation to answer that question, collect the **D**ata, **A**nalyse the data, draw a **C**onclusion and write a report.

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The Statistical Reasoning Learning Progression

Zone 1	*Reads a single value from a simple graph on either the x or y axis but does not look at axes values simultaneously. Tends to focus on the highest values unless directed. Variation is considered visually but not otherwise recognised. Is familiar with standard simple probabilistic/chance situations (e.g., dice, coins) but uses these at a superficial level. Can give real world examples of variation (e.g., weather).*
Zone 2	*Uses reasoning that recognises variation in some way but may not appreciate expectation (chance). Uses the language of 50% or 50:50 but does not appreciate the meaning. Reads information from simple graphs using x and y axes, and can describe what the graph is about but may not recognise association between variables. Is familiar with simple chance experiments but is unable to reason quantitatively and is likely to rely on personal beliefs rather than the data when explaining an outcome (e.g., it's the way you roll the die). Recognises one aspect of sampling, such as size or method, in a familiar context but does not coordinate these ideas to provide a random and representative sample.*
Zone 3	*Recognises expectation but interprets this in terms of strict probability or based on a visual representation. Recognises variation but in graphing situations may explain this based only on visual representation rather than quantitative reasoning. Applies ideas of variation drawing on expectation but only in familiar contexts such as coin flips. Recognises expectation in statistical situations (e.g. value of mean/median) but explanations are limited. Reasons quantitatively using direct comparison but relies on additive thinking (e.g., not recognising proportion. In more complex inference tasks or less familiar contexts, draws on opinion rather than data or retreats to "luck" as an explanation in more complex situations.*
Zone 4	*Compares data in two graphs but focuses on single elements only. Can associate two variables with a single value and provides descriptive explanations. Recognises variability and expectation in more complex random situations but explanation refers to uncertainty in general terms and is not quantified or is based on strict probability (expectation). May not recognise the importance of equal likelihood. Recognises relative order in the language of uncertainty but does not appreciate some subtleties. Reasons quantitatively in familiar situations involving related comparisons and in the context of uncertainty. Relies on additive thinking in situations involving measures of central tendency, and is unlikely to question the quality of data. Critiques sampling approaches using single aspects only (i.e., size or method) in an evaluative situation. Falls back on personal beliefs in more complex situations when asked for an explanation.*
Zone 5	*Provides a statistical explanation, but this may be incomplete, and can recognise important information in making comparisons. Intuitively suggests association expressed in non-quantitative ways and recognises equal likelihood. Recognises appropriate sample size and provides appropriate critiques of sampling method but does not explicitly include randomness. Recognises simple proportion in chance contexts and orders language of chance qualitatively. Recognises relevant aspects of graphical representation and uses these to reason statistically but may not include all aspects. Recognises key aspects of central tendency but reverts to non-statistical justifications. Implicitly recognises that all combinations of numbers have the same chance of appearing when randomly sampled from a grid with no replacement.*
Zone 6	*Interprets and describes the association between two variables and considers the implication in visual contexts. Beginning to work with the association between two variables in a non-graphical format. Constructs reasonable arguments based on an understanding of chance and probability and context. Provides a sensible critique of sampling in relation to method and sample size but is implicit rather than explicit about randomness. Uses measures of central tendency to justify a closed response.*
Zone 7	*Recognises and describes the spread of data explicitly in a statistical sense using statistically important information and not just a visual image. Recognises variation appropriately including ideas of fairness, equality of outcomes, and distribution as appropriate. Uses all available data to justify decisions or evaluations statistically. Makes reasonable sampling decisions that recognise the importance of randomness and critiques inappropriate non-representative and/or non-random samples. Uses data to justify responses and recognises limitations but may still revert to offering an*

	opinion based on individual beliefs. Recognises variability relative to context and the nature of a distribution to provide a realistic solution.
Zone 8	*Recognises variability relative to context using proportional reasoning to support arguments. Can recognise, coordinate, and integrate all relevant information to make evidence-based decisions using proportional reasoning and relevant context. Applies ideas about central tendency to justify explanations and decisions and is able to make appropriate statistical critiques of sampling using size, method, range, representativeness in justification. Recognises equal chance and interprets chance situations mathematically rather than offering an opinion.*