Reframing Mathematical Futures II

Geometric Reasoning Learning Progression - Teaching Advice

There are three pillars that support all of geometric reasoning. These have been identified as Visualisation, Language and Discourse, and Representations. These three pillars are intertwined and are integral to all teaching and learning in the area of Geometry.

- **Visualisation**

This includes imagining the shapes and objects and manipulating them in one's mind, picturing what might be seen as the shape/object is rotated, or reflected, stretched or shrunk, or moved to a different position in relation to other shapes/objects, and orientation and location. It is about looking for patterns and relationships and interpreting and reconstruction what is found in the light of existing knowledge. Of particular relevance to measurement is being able to "see" how a shape might be cut and the pieces rearranged to form another shape.

- **Language and Discourse**

This includes all of the language and discussion, encompassing both the common vocabulary used to describe specific mathematical properties, the formal definitions of terms and all other forms of communication such as diagrams and symbols. Language and participation in discussion using description, explanation and justification are really important in the development of all mathematical concepts and in being able to reason.

- **Representations**

This includes photographs, drawings, different diagrammatic ways of representing objects and mathematical ideas, and construction of shapes/objects in 2D format or from different materials. Of particular relevance is seeing multiple representations and making connections between them.

The field of geometry is wide and covers many aspects. In considering the Learning Progression and Teaching Advice for Geometry three big ideas that rested on these pillars were identified as Hierarchy and Properties, Transformation of Relationships and Geometric Measurement. These have been used to assist in structuring the Learning Progression and Teaching Advice, but it is also important to recognise that there are strong connections between all three and they do not stand alone.

- **Hierarchy and Properties**

Hierarchy and Properties is strongly based on the notion of all geometric objects, whether in two dimensions or three dimensions, being defined by some of their properties and having other properties which are often used in applications and reasoning. Within this is the idea that a shape or object has a range of properties, hence classification may place it in many groups of shapes or objects and these form a hierarchy.

- **Transformation of Relationships**

Geometric objects can be transformed through operations such as rotation and dilation and in these transformations some relationships may stay the same or change. Within this is the idea that objects and

groups of objects appear different when viewed from different perspectives. Location and the connectedness of the space is also part of transformation of relationships.

- **Geometric Measurement**

Geometry is concerned with the space around us and geometric measurement is an idea at the core of exploring the objects in this space. Geometric measurement is a process of quantifying the world with measurements being represented by a number and a unit. Reasoning about measurement situations necessitates an understanding of measurement attributes such as length, area, volume and mass, knowledge of units and unit composition, and application of these knowledge in problem solving situations. Key ideas arising include estimation, tolerance of errors, scale, creation and use of measuring implements, unit and the choice of unit. These are all closely related to each other and to other aspects of geometry and are built on the three pillars of reasoning.

ZONE 1 Description	Teaching Implications
Hierarchy and properties *Recognise shapes by appearance and common orientation,* ***Transformation of Relationships*** *Shows emerging recognition of objects from different perspective* *Show emerging recognition of reflectional symmetry of objects and shapes* *Show emerging recognition of a coordinate system.* ***Geometric Measurement*** *Understand the attribute of length, area and mass in terms of comparison*	**Consolidate and Establish** ➢ **2D –** Experience with a large range of different shapes, particularly non-prototypical ones. ➢ **3D –** Name common 3D objects, identify some of the features in terms of faces, vertices and edges ➢ **Transformation and Location –** Exploring different perspectives on objects and collections of objects (e.g. bird's or spider's eye view of classroom from own perspective) Experience identifying symmetry of shapes and patterns with mirror lines in different positions and in creating symmetric patterns Directional language of left, right, top bottom Use of a coordinate system (street map) to identify locations and give directions ➢ **Measurement –** Ordering physical objects based on attributes of length, area and mass **Introduce and Develop** ➢ **2D –** Shapes in environment using language associated with describing simple shapes/objects and their properties to justify their identification ➢ **3D –** Faces of solids and deconstruction of solids to nets leading to understanding how to create nets from solid objects and recreate objects from the nets Name a range of 3D objects and identify some of their features (e.g. square faces on cube) Draw simple 3D objects so that the features are identifiable. Draw perspective views of 3D objects ➢ **Transformation and location –** Symmetry adding mirror lines, moving to accepting more than one symmetry. Rotational symmetry as well as reflectional symmetry Informal tessellations of shapes Identification of 3D objects from perspective views (e.g. top, front and right view of a prism) Identification of features on alphanumeric grid maps and both following and giving directions ➢ **Measurement –** Estimation of length measure, Introduction of angle as a measure of turn Informal units used to order objects by length, area, angles and mass Identification of quarter turns, half turns, 3 quarter turns, full turns, clockwise, anticlockwise. Comparison and ordering of angle size

ZONE 2 Description	Teaching Implications
Hierarchy and properties *Identifies simple shapes in situ and on simple solids* *Identifies standard nets of simple solids* *Beginning to represent 3D objects and using some related language;* *Informed awareness of some properties that discriminate shapes* *Some language beginning but cannot coordinate information /manipulate/check sufficiency of information* ***Transformation of Relationships*** *Recognises some reflection symmetry,* *Appreciates/imagines 3D shapes but limited ability to mentally manipulate images (e.g. rotate shapes),* *Can sometimes see an object from a different perspective but cannot justify thinking.* *Limited capacity to identify directions* ***Geometric Measurement*** *Shows emerging understanding of measurement concepts such as length, area, mass and angle.* *Uses informal units of measurement for comparison.* *Understand angle as a measure of turn.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Geometric properties of shapes used when discussing and justifying choice of shape names in different contexts and explanation of reasoning about shape identification (e.g. feely box) Identification of shapes presented in non-standard orientations with one or two specific properties Construction of specific shapes with compass and straight edge and/or Geogebra using properties ➢ **3D** – Properties relating to 3D objects using concrete materials and informal/formal language, and description of different perspectives views including both drawing objects from different perspectives and building objects from different perspective drawings ➢ **Transformation and location** – Identification of 2D shapes that have been transformed under simple reflections and rotations Coordinates related to maps and Cartesian coordinates in first quadrant Identification of items on maps from coordinates and placing items on map given coordinates for both street and Cartesian maps ➢ **Measurement –** Order shapes and objects by area and volume with justification of choices Situations in measurement where different measurement attributes are considered - length/width/height and area and volume. For example, if the area is 12 square tiles, what might be the distance around the shape? Recognition and identification of specific angles such as right angle, straight angle and reflex angle.

ZONE 3 Description	Teaching Implications
Hierarchy and properties *Uses one or two properties (insufficient) to explain their reasoning about shapes (e.g. triangles and quadrilaterals)* *Beginning to coordinate multiple information sources, but not yet justifying (only using part of information, e.g. check net to see if it will make a cube)* *Can make and name 2D shapes (simple), not recognising right angles, parallel lines, and not recognising properties in non-standard representations* *Limited capacity to represent 3D objects* ***Transformation of Relationships*** *Able to visualise some objects from different perspective* *Able to use coordinates in first quadrant* *Beginning to manipulate visual images and coordinate information* ***Geometric Measurement*** *Uses one or two attributes (in sufficient) to explain their reasoning about measurement (e.g. consider length but forget impact of width/height)* *Beginning awareness of volume and capacity and the relationship between length, area and volume).*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Identification of all properties of 2D shapes such as types of triangles and quadrilaterals. Recognition of parallel and perpendicular lines and right angles in situ Identification of shapes from sets of properties (e.g. has 2 right angles and at least one pair of parallel lines) Development of language such as diagonal, perpendicular, rotation and regular and encouragement of justification ➢ **3D** – Families of polyhedron and identification of features that relate to the names (e.g. prisms and pyramids) Comparison of families of shapes/objects, describing what is the same and what is different A variety of representations of 3D objects including nets, isometric and perspective drawings Construction of 3D objects from isometric drawings or other representations. Identification of nets of simple 3D objects from drawings of joined 2D shapes ➢ **Transformation and location** – Transformation of 2D shapes under simple reflections and rotations Giving directions from one position to another on a map using different routes and using both perspective and directional approaches (e.g. go 2km turn right or go 2km south turn west). ➢ **Measurement –** Formal units of length and use of them in calculation of perimeter, area and volume in open and/or real situations explaining solutions Volume and capacity using standard units Relationships between length, perimeter, area and volume Identification of relevant information required to answer specific questions given measurement situations with a range of information. Order, estimate and measure angles Deduce angle sizes in simple situations (e.g. folding square diagonally to produce a 45° angle).

ZONE 4 Description	Teaching Implications
Hierarchy and properties *Recognises relevance of properties in more complex shapes and able to use some of the language but unable to use those properties or only focuses on one aspect.* *Recognises some conditions for a shape (e.g. square), but not dealing with all relevant information and cannot explain reasoning and unable to recognise minimum and sufficient conditions.* *Knows names of some 3D objects (difference between prism and pyramids.,* *Identifies 3D objects from 2D formats such as isometric drawing and nets.* *Generally incomplete reasoning in geometric situations.* ***Transformation of Relationships*** *Able to visualise objects from different perspective but limited capacity to explain (GPERS3.1).* *Give directions of a map from personal rather than other viewer's perspective.* *Recognise same/different in transformation situations.* ***Geometric Measurement*** *Performs measurement calculations but attends to only one attribute.* *Recognises relevant aspects in informal measurement context, (e.g. GSOD 1.2) but in one aspect (linear measure) only.* *Incomplete reasoning in measurement situations.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Similarities and differences between shapes; including geometric concepts of similar and congruent Inclusion of angle and diagonal properties in identification of 2D shapes using properties with justification. Identification of classes of triangles and quadrilaterals using properties. Given one or two properties, identification of all possible types of shapes Description of all properties of a family of shapes Reasoning about the veracity of geometric hypotheses (e.g. all triangles tessellate) ➢ **3D** – Identification of possible 3D objects from a group of properties (e.g. What shapes can you make if you have 2 equilateral triangles, 2 squares and 4 rectangles?; I have 3 pairs of parallel faces, and 6 faces, what am I?) Creation of the nets of simple 3D objects Identification of all nets of a simple 3D object ➢ **Transformation and location** – Use of proportional reasoning to dilate shapes; calculation of the new measurements from those of the original shape (some should include coordinates) Description of all properties of a family of objects Symmetry of shapes including both reflectional and rotational symmetry Simple reflections and rotations on a coordinate grid Coordinates in all four quadrants ➢ **Measurement –** Relationships between length, perimeter and area. Angles in polygons (including sum of interior angles Deduction of angle size in situations involving parallel lines and transversal and in polygons. Application of measurement knowledge to solving problems in real life situations (e.g. length of edging needed for garden bed, area of garden bed, etc)

ZONE 5 Description	Teaching Implications
Hierarchy and properties *Uses properties or orientations to reason in geometric situation. Recognises parallel lines in non-standard representation, access to relevant language, able to recognise and use appropriate information to solve problem. Identifies and recognises relevance of multiple representation 2D, beginning to use but not recognise sufficient condition, unable to recognise redundancy**.*** *Use more complex language but cannot use to explain, can use in specific context.* ***Transformation of Relationships*** *Able to visualise and represent 3D objects in 2D (i.e. Nets). Demonstrates knowledge of dilation and coordinate systems. Uses landmarks but retains personal orientation when providing directions, have some insight into rotational symmetry.* ***Geometric Measurement*** *Provides partial solutions and explanations when calculating measurement situations.* *Uses geometrical knowledge to find some angles but unable to explain.* *Coordinating multiple information but not necessary identifying, attending to, or recognising, what is not needed, how it is relevant to context, (e.g. not recognise box too big)*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Identification of shapes from properties with rationales Similarity and differences between classes of shapes Given one or two properties, deduce other properties (e.g. A shape has 4 equal side, what other properties must it have and what other properties might it have?) Construct own understanding of the hierarchy of quadrilaterals Construct arguments based on properties of shapes ➢ **3D** – Identification of classes of 3D objects from their properties Regular polyhedron (platonic solids) ➢ **Transformation and location** – Identification of mirror lines and points of rotation that are both within and outside shapes (including coordinate system) Rotation of 2D shapes and identification of rotational symmetry ➢ **Measurement –** Reason about the sum interior angles of polygons Measurement calculation problems which require multi-steps Relationships between volumes of simple cuboids and dimensions and surface area Application of surface area and volume with simple prisms Angle magnitude and relationships between angle sizes in a variety of angle situations such as circles, polygons…

ZONE 6 Description	Teaching Implications
Hierarchy and properties *Uses properties accurately when reasoning about spatial situations but lack knowledge of geometry hierarchy**.*** *Understands properties 2D shapes but not special cases (e.g. regular).* *Geometric arguments rely on examples/counter examples* ***Transformation of Relationships*** *Recognises line symmetry but may over generalise and apply inappropriately.* *Provides accurate directions from a map, using appropriate language and able to describe directions from walker's perspective.* ***Geometric Measurement*** *Understands impact of doubling dimensions on volume, able to visualise volume and calculate when numbers are small.* *Able to make deduction about angle situations with limited explanations,* *Measurement arguments rely on examples/counter examples* *Omit one step when calculating multi-step measurement problems.* *Beginning to reason deductively.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Identification of necessary and sufficient conditions (e.g. Given a shape has 4 equal sides, what other property is needed to ensure that it is a square?) Proving the properties of shapes based on concepts of similarity and congruence (e.g. Given that a quadrilateral's diagonals bisect the angles, prove that it is a rhombus). Hierarchical structure of quadrilaterals Use own words to write definitions of shape and properties ➢ **3D** – Cross sections of 3D objects (e.g. identifying faces of cross section cuts). ➢ **Transformation and location** – Isometric transformations of shape in a coordinate system Non-isometric transformation of simple 2D shapes, particularly on a coordinate grid ➢ **Measurement –** Area of polygonal and circle shapes presented in non-prototype orientation (e.g. shapes presented on coordinate grid). Relationships between surface area and volume Use of knowledge of 2D shapes, similarity and congruence to determine the size of angles and make justifications Application of measurement principles and calculations to complex shapes that require partitioning of shapes with explanation/justification of solutions

ZONE 7 Description	Teaching Implications
Hierarchy and properties *Can recognise necessary and sufficient conditions for all triangle families of shapes.* *Working analytically with properties of rectangles, recognising necessary and sufficient conditions for rectangle and square.* *Recognises and uses concepts of similarity and congruence in argument.* *Beginning to recognise necessary and sufficient conditions, use sound reasoning in argument/explanations* ***Transformation of Relationships*** *Can apply isometric transformations and simple dilations to 2D shapes using a variety of platform including the coordinate systems.* ***Geometric Measurement*** *Explanations often procedurally based.* *Able to recognise the relationship between length, area and volume.* *Able to calculate area of shapes on a coordinate system.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **2D** – Create own diagrams showing the hierarchy of quadrilaterals and other families of 2D shapes. Construction of different types of quadrilaterals and other polygons (e.g. using platforms such as compass and ruler construction and Geogebra). Use principles of geometric properties to reason and justify conjecture about properties of shapes (e.g. join the midpoints of a rectangle, make a conjecture about the shape formed and prove it; All quadrilaterals tesselate the plane) ➢ **3D –** Develop arguments about 3D objects (e.g. agree or disagree with the statement… giving justification for your position [There can only be 5 regular polyhedra]) ➢ **Transformation and location** – Sequential transformations on a shape in a coordinate system Construct a variety of shapes under particular reflectional and rotational conditions (e.g. draw a shape that has exactly two lines of symmetry and a rotational symmetry of 2; construct a shape that has rotational symmetry of 3 and no line of symmetry) ➢ **Measurement –** Use measurement in a variety of 2D/3D problem situations including angles in a circle, Pythagoras and trigonometry to solve problems with justification of solutions

ZONE 8 Description	Teaching Implications
Hierarchy and properties *Can recognise necessary and sufficient conditions for all quadrilateral families of shapes and polygons and circles.* *Constructs arguments based on multiple properties of 2D shapes and 3D objects.* *Uses the necessary and sufficient conditions to reason about geometric situations, conjectures and propositions (theorem.)* ***Transformation of Relationships*** *s understanding of both reflectional and rotational symmetry.* ***Geometric Measurement*** *Uses concept in measurement clearly to argue about conjectures, propositions and real situations.*	**Consolidate and Establish** ➢ What was Introduced and Developed in previous Zone **Introduce and Develop** ➢ **Proof.** What constitutes proof? Explore the use of proof in a variety of geometric and measurement situations ➢ **Hierarchy**. Extend hierarchy ideas to 3D looking at the properties of 3D objects, and in particular, polyhedra. How are angles seen in 3D objects? What is meant by the angle between two faces? How do different names reflect properties? For example a cube is also a hexahedron. It is a regular hexahedron. How do these ideas of regular and properties fit in a 3D context? ➢ **Transformation**. With particular reference to 3D objects explore cross sections and the drawing of cross sections of objects. Investigate symmetry of 3D objects. On a coordinate grid explore the effects of different transformations (both isometric and non-isometric) on polygons drawn on the grid and consider the effect of the transformations on the areas of these shapes ➢ **Measurement**. Extend measurement to more complex 2D/3D situations including packing problems and surface area and volume problems. ➢ Apply measurement in more complex real problem contexts including surveying and investigations of buildings and structures.