Reframing Mathematical Futures II

Geometric Reasoning Learning Progression

There are three pillars that support all of geometric reasoning. These have been identified as Visualisation, Language and Discourse, and Representations. These three pillars are intertwined and are integral to all teaching and learning in the area of Geometry.

- **Visualisation**

This includes imagining the shapes and objects and manipulating them in one's mind, picturing what might be seen as the shape/object is rotated, or reflected, stretched or shrunk, or moved to a different position in relation to other shapes/objects, and orientation and location. It is about looking for patterns and relationships and interpreting and reconstructing what is found in the light of existing knowledge. Of particular relevance to measurement is being able to "see" how a shape might be cut and the pieces rearranged to form another shape.

- **Language and Discourse**

This includes all of the language and discussion, encompassing both the common vocabulary used to describe specific mathematical properties, the formal definitions of terms and all other forms of communication such as diagrams and symbols. Language and participation in discussion using description, explanation and justification are really important in the development of all mathematical concepts and in being able to reason.

- **Representations**

This includes photographs, drawings, different diagrammatic ways of representing objects and mathematical ideas, and construction of shapes/objects in 2D format or from different materials. Of particular relevance is seeing multiple representations and making connections between them.

The field of geometry is wide and covers many aspects. In considering the Learning Progeression and Teaching Advice for Geometry three big ideas that rested on these pillars were identified as Hierarchy and Properties, Transformation of Relationships and Geometric Measurement. These have been used to assist in structuring the Learning Progression and Teaching Advice, but it is also important to recognise that there are strong connections between all three and they do not stand alone.

- **Hierarchy and Properties**

Hierachy and Properties is strongly based on the notion of all geometric objects, whether in two dimensions or three dimensions, being defined by some of their properties and having other properties which are often used in applications and reasoning. Within this is the idea that a shape or object has a range of properties, hence classification may place it in many groups of shapes or objects and these form a hierarchy.

- **Transformation of Relationships**

Geometric objects can be transformed through operations such as rotation and dilation and in these transformations some relationships may stay the same or change. Within this is the idea that objects and

groups of objects appear different when viewed from different perspectives. Location and the connectedness of the space is also part of transformation of relationships.

- **Geometric Measurement**

Geometry is concerned with the space around us and geometric measurement is an idea at the core of exploring the objects in this space. Geometric measurement is a process of quantifying the world with measurements being represented by a number and a unit. Reasoning about measurement situations necessitates an understanding of measurement attributes such as length, area, volume and mass, knowledge of units and unit composition, and application of these knowledge in problem solving situations. Key ideas arising include estimation, tolerance of errors, scale, creation and use of measuring implements, unit and the choice of unit. These are all closely related to each other and to other aspects of geometry and are built on the three pillars of reasoning.

The Geometrical Reasoning Learning Progression

Zone 1	*Pre-cognition - Recognise simple shapes by appearance and common orientation; show emerging recognition of objects from different perspectives; naming and describing 3D objects base on common 2D shape names; identifying some standard nets; identifying location using simple referencing system. In measurement situations, recognising comparisons in 1dimension without using units.*
Zone 2	*Recognition - Identify simple shapes in situ and on simple solids; recognise some reflective symmetry, nets of simple solids and simple shapes. Show emerging representation of 3D objects; use of language of geometry and emerging perception of measurement concepts such as length, area, and angle but do not coordinate information or justify thinking. Beginning to represent and move between representations, focuses mostly on one property.*
Zone 3	*Emerging informal reasoning - Use one or two properties or attributes (insufficient) to explain their reasoning about shapes and measurement but often do not recognise properties in non-standard representations. Demonstrate a beginning understanding of measurement attributes. Tend to visualise objects from own perspective. Use simple coordinates. Tend to see objects and groups of objects as a whole but unable to analyse components independently.*
Zone 4	*Informal but insufficient reasoning - Know some geometric language, name some 3D objects and able to visualise objects from a different perspective but show incomplete reasoning in geometric and measurement situations, attending to necessary but not sufficient properties and using properties to identify shapes/objects. Perform measurement calculations but attend to only one attribute. Give directions from a map from personal rather than other viewer's perspective when situations are more complex.*
Zone 5	*Emerging analytical reasoning - Able to visualise and represent 3D objects using 2D platform (Nets); recognise properties in non-standard orientations and starting to use properties to identify classes; beginning to use but not recognise sufficient conditions; use either properties or orientations to reason in geometric situation; access relevant geometric language; demonstrate knowledge of dilation and coordinate systems and recognises some rotational symmetry; use landmarks but retain personal orientation when providing directions; provide partial solutions and explanations when calculating measurement situations. Begin to coordinate multiple components.*
Zone 6	*Property-based analytical reasoning - Use properties accurately when reasoning about spatial situations but lack knowledge of geometry hierarchy. Understand properties 2D shapes but not special cases (e.g. regular). Geometric and measurement arguments rely on examples/counter examples. Provide accurate directions from a map using appropriate language and able to describe directions from walker's perspective. Understand impact of doubling dimensions on volume, able to visualise volume and calculate when numbers are small. Omit one step when calculating multi-step measurement problems. Able to make deduction about angle situations with limited explanations. Beginning to reason deductively but not able to coordinate all aspects.*
Zone 7	*Emerging deductive Reasoning - Work analytically with properties of rectangles. Beginning to recognise necessary and sufficient conditions. Use sound reasoning in argument/explanations, though explanations often are procedurally based or base on an example. Able to recognise the relationship between length, area and volume. Using multiple properties to reason but in measurement situations may rely on procedural explanations.*
Zone 8	*Logical, inference-based reasoning - Construct arguments based on multiple properties of 2D shapes and 3D objects, using the necessary and sufficient conditions to reason about geometric and measurement situations, conjectures and propositions (theorem), demonstrate understanding of both reflectional and rotational symmetry.*