Reframing Mathematical Futures II

Algebraic Reasoning Learning Progression

Three big ideas that underpin algebraic reasoning have been identified as Pattern and Function, Equivalence, and Generalisation. These big ideas are not discrete, but are intertwined. The Algebraic Learning Progression and the associated Teaching Advice have been structured around these three big ideas that are briefly described here.

- **Pattern and Function**

Pattern and Function is strongly based on noticing structure. Included within this big idea is noticing the structure of patterns, including number patterns and the structure of arithmetic. Within Pattern and Function, is the identification of relationships and inverse relationships, as well as the identification of variables and constants. Being able to move flexibly between multiple representations is an important aspect, as are the ideas of continuous and discrete functions, rates of change and families of functions.

- **Equivalence**

This big idea has at its core the notion of balance or balancing, including both known and unknown terms. Equivalence includes being able to understand the meaning of the equals sign, using comparative language, transforming and identifying equivalent expressions, especially in the solution of equations and inequations. The development of relational thinking is an important aspect. This includes using the four operations to produce equivalent numerical expressions, by recognising and creating equivalent sums, differences, products and quotients, and using equivalence to simplify calculations. In using symbolic expressions, students know how to combine two (or more) equivalent symbolic expressions retaining equivalence.

- **Generalisation**

Generalisation strongly connects arithmetic and algebraic thinking by focussing on similarities and differences. It relies on the noticing of structure. Generalisation includes moving from the specific to the general and moving from the general to the specific. This includes a variety of representations, such as models, words, pictures, graphs and symbols. The structure of arithmetic, number and algebraic laws, patterns, functions and equivalence situations may offer opportunities to develop generalisation. The meaningful use of mathematical language and symbols may also be developed through the process of generalisation.

The Algebraic Reasoning Learning Progression

Zone 1	*Can continue simple patterns, but is likely to build them additively. Reasoning is confined to specific incidences and numerical examples of simple physical situations. Arithmetic thinking is used. Abstraction and generalisation not evident at this stage.*
Zone 2	*Beginning to recognise patterns and relationships and conjecture about these. Able to identify numbers that vary and numbers that stay the same. Engages with the context, but arithmetic reasoning, typically based on calculations, is still being used. Recognises some multiples and some relationships like 6 more/6 less, while not necessarily recognising equivalence. Can work with simple scales and transfer from a table of values to a graph.*
Zone 3	*Beginning to use symbolic expression and elementary reasoning. While still using arithmetic approaches there is evidence of relational reasoning with the numbers and providing some explanation. Beginning to recognise simple multiplicative relationships. There is some evidence of co-ordination of two ideas. Explanation and justification is limited. Algebraic expressions are used rather than equations. Beginning to recognise equivalent relationships. Can explain simple generalisations by telling stories, manipulating materials and very simple use of symbolic language.*
Zone 4	*Beginning to work multiplicatively and simultaneously co-ordinate variables, although still uses specific examples to convince. Able to reason and generalise in simple situations. Can recognise and interpret the relevance of range from table and/or graphs and to recognise functional relationships. When faced with more complex algebraic situations is unable to use the full range of explanation or handle all of the information simultaneously. Beginning to transition to abstraction by inserting a number for a pronumeral.*
Zone 5	*Able to use multiplicative reasoning in simple situations. Can reason with more complex additive situations involving larger numbers and subtraction but usually by examples. Has moved from algebraic expressions to using equations. Can derive a strategy that maintains equivalence, but cannot yet generalise the situation. Able to use symbols to express rules. Can follow, compare and explain rules for linking successive terms in a sequence. Recognises and represents simple functional representations. Can justify an argument using mathematical text. Beginning to generalise using words or using some symbolic generalisations in simple situations, usually building on in context.*
Zone 6	*Can use and interpret basic algebraic conventions to represent situations involving a variable quantity. Beginning to explain using logical language and to use if … then reasoning. Uses symbolic language but the need for simplification is still being developed. Able to generalise arithmetic relationships with justification, including simple multiplicative relationships, but are often still context bound. Can show why several expressions are equivalent, typically employing numerical (non-symbolic) justifications*
Zone 7	*Is able to use and interpret algebraic conventions for representing generality and relationships between variables. Beginning to use sound logical reasoning with appropriate reasoning language (e.g. if … then, must) evident. There is more co-ordination of multiplicative thinking and the associated language to notice algebraic structure. Can recognise and use the relationships between multiple entities and connections between and within different representations. Is able to establish and describe equivalence explaining relationships using the distributive property and the inverses of addition and multiplication. Can generalise quite complex situations and in more direct situations is beginning to use simplest form.*
Zone 8	*Is able to combine a facility with symbolic representation and an understanding of algebraic concepts to represent and explain mathematical situations. Explanations are sophisticated using logical thought and the language of reasoning. Can use multiple representations in a co-ordinated manner to solve, analyse, convince and conclude. Can visualise the form and structure of a function, at least graphically, from a real context. Is able to work in a context free environment using symbolic language and treat algebraic expressions (e.g. 3x + 2) as single entities. Can generalise more complex situations. Is able to establish and describe equivalence involving the four operations explaining relationships in symbolic terms. Can use abstract symbols to solve problems in context with multiple steps.*