## Reframing Mathematical Futures II

# Geometric Reasoning Learning Progression - Teaching Advice

There are three pillars that support all of geometric reasoning. These have been identified as Visualisation, Language and Discourse, and Representations. These three pillars are intertwined and are integral to all teaching and learning in the area of Geometry.

- **Visualisation**

This includes imagining the shapes and objects and manipulating them in one's mind, picturing what might be seen as the shape/object is rotated, or reflected, stretched or shrunk, or moved to a different position in relation to other shapes/objects, and orientation and location. It is about looking for patterns and relationships and interpreting and reconstructiong what is found in the light of existing knowledge. Of particular relevance to measurement is being able to "see" how a shape might be cut and the pieces rearranged to form another shape.

- **Language and Discourse**

This includes all of the language and discussion, encompassing both the common vocabulary used to describe specific mathematical properties, the formal definitions of terms and all other forms of communication such as diagrams and symbols. Language and participation in discussion using description, explanation and justification are really important in the development of all mathematical concepts and in being able to reason.

- **Representations**

This includes photographs, drawings, different diagrammatic ways of representing objects and mathematical ideas, and construction of shapes/objects in 2D format or from different materials. Of particular relevance is seeing multiple representations and making connections between them.

The field of geometry is wide and covers many aspects. In considering the Learning Progeression and Teaching Advice for Geometry three big ideas that rested on these pillars were identified as Hierarchy and Properties, Transformation of Relationships and Geometric Measurement. These have been used to assist in structuring the Learning Progression and Teaching Advice, but it is also important to recognise that there are strong connections between all three and they do not stand alone.

- **Hierarchy and Properties**

Hierachy and Properties is strongly based on the notion of all geometric objects, whether in two dimensions or three dimensions, being defined by some of their properties and having other properties which are often used in applications and reasoning. Within this is the idea that a shape or object has a range of properties, hence classification may place it in many groups of shapes or objects and these form a hierarchy.

- **Transformation of Relationships**

Geometric objects can be transformed through operations such as rotation and dilation and in these transformations some relationships may stay the same or change. Within this is the idea that objects and

groups of objects appear different when viewed from different perspectives. Location and the connectedness of the space is also part of transformation of relationships.

- **Geometric Measurement**

Geometry is concerned with the space around us and geometric measurement is an idea at the core of exploring the objects in this space. Geometric measurement is a process of quantifying the world with measurements being represented by a number and a unit. Reasoning about measurement situations necessitates an understanding of measurement attributes such as length, area, volume and mass, knowledge of units and unit composition, and application of these knowledge in problem solving situations. Key ideas arising include estimation, tolerance of errors, scale, creation and use of measuring implements, unit and the choice of unit. These are all closely related to each other and to other aspects of geometry and are built on the three pillars of reasoning.

| ZONE 1 Description | Teaching Implications |
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| *Recognise simple shapes by appearance and common orientation; show emerging recognition of objects from different perspectives; naming and describing 3D objects base on common 2D shape names; identifying some standard nets; identifying location using simple referencing system. In measurement situations, recognising comparisons in 1dimension without using units.*<br><br>***Hierarchy and properties***<br>*Recognise shapes by appearance and common orientation,*<br><br>***Transformation of Relationships***<br>• *Show emerging recognition of objects from different perspective*<br>• *Show emerging recognition of reflectional symmetry of objects and shapes*<br>• *Show emerging recognition of a coordinate system.*<br><br>***Geometric Measurement***<br>*Understand the attribute of length, area and mass in terms of comparison* | **Consolidate and Establish**<br>➢ **2D** - Experience with a large range of different shapes, particularly non-prototypical ones.<br>➢ **3D** - Name common 3D objects, identify some of the features in terms of faces, vertices and edges<br>➢ **Transformation and Location** –<br>Exploring different perspectives on objects and collections of objects (e.g. bird's or spider's eye view of classroom from own perspective)<br><br>Experience identifying symmetry of shapes and patterns with mirror lines in different positions and in creating symmetric patterns<br><br>Directional language of left, right, top bottom<br>Use of a coordinate systems (street map) to identify locations and give directions<br><br>➢ **Measurement –** Ordering physical objects based on attributes of length, area and mass<br><br>**Introduce and Develop**<br>➢ Language associated with describing simple shapes/objects and their properties<br>➢ Rotational symmetry as well as reflectional symmetry<br>➢ Exploring the faces of solids and deconstructing solids to nets<br>➢ Using informal measures to compare (length, angles)<br>➢ Estimating length measure, introduce angle as a measure of turn |

| ZONE 2 Description | Teaching Implications |
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| *Identify simple shapes in situ and on simple solids; recognise some reflective symmetry, nets of simple solids and simple shapes. Show emerging representation of 3D objects; use of language of geometry and emerging perception of measurement concepts such as length, area, and angle but do not coordinate information or justify thinking. Beginning to represent and move between representations, focuses mostly on one property.*<br><br>***Hierarchy and properties***<br>• *Identify simple shapes in situ and on simple solids*<br>• *Identify standard nets of simple solids*<br>• *Beginning to represent 3D objects and using some related language;*<br>• *Informed awareness of some properties that discriminate shapes*<br>• *Some language beginning but cannot coordinate information /manipulate/check sufficiency of information*<br><br>***Transformation of Relationships***<br>• *Recognise some reflection symmetry,*<br>• *Appreciate/imagine 3D shapes but limited ability to mentally manipulate images (eg. rotate shapes),*<br>• *Can sometimes see an object from a different perspective but cannot justify thinking.*<br>• *Limited capacity to identify directions*<br><br>***Geometric Measurement***<br>• *Show emerging understanding of measurement concepts such as length, area, mass and angle.*<br>• *Use informal units of measurement for comparison.*<br>• *Understand angle as a measure of turn.* | **Consolidate and Establish**<br>➢ **2D** – Explore shapes in environment using language to justify their identification<br>➢ **3D** –<br>Create nets from solid objects and recreate objects from the nets<br><br>Name a range of 3D objects and identify some of their features (e.g. square faces on cube)<br><br>Draw simple 3D objects so that the features are identifiable.<br><br>Draw perspective view of 3D objects<br><br>➢ **Transformation and location** –<br>Identify symmetry adding mirror lines, need to move beyond and accept more than one symmetry<br><br>Informal tessellations of shapes<br><br>Identify 3D objects from perspective views (e.g. top, front and right view of a prism)<br><br>Use maps to identify features on the alphanumeric grid and both follow and give directions<br><br>➢ **Measurement** –<br>Using informal units to order objects by length, area, and mass<br><br>Identify quarter turns, half turns, 3 quarter turns, full turns, clockwise, anticlockwise.<br><br>Compare and order angle size<br><br>**Introduce and Develop**<br>➢ Use geometric properties of shapes when discussing and justifying their choice of shape names<br>➢ Describe all the properties relating to 3D objects using concrete materials and informal/formal language, and describe what they see from different perspectives<br>➢ Introduce the idea of coordinate related to maps and Cartesian coordinates in first quadrant<br>➢ Explore situations in measurement where both length/width/height and area and volume are considered. For example, if the area is 12 square tiles, what might be the distance around the shape? |

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| **ZONE 3 Description** | **Teaching Implications** |
| *Use one or two properties or attributes (insufficient) to explain their reasoning about shapes and measurement but often do not recognise properties in non-standard representations. Demonstrate a beginning understanding of measurement attributes. Tend to visualise objects from own perspective. Use simple coordinates. Tend to see objects and groups of objects as a whole but unable to analyse components independently.*<br><br>***Hierarchy and properties***<br>• *Use one or two properties (insufficient) to explain their reasoning about shapes (e.g. triangles and quadrilaterals)*<br>• *Beginning to coordinate multiple information sources, but not yet justifying (only using part of information, e.g. check net to see if it will make a cube)*<br>• *Can make and name 2D shapes (simple), not recognising right angles, parallel lines, and not recognising properties in non-standard representations*<br>• *Limited capacity to represent 3D objects*<br><br>***Transformation of Relationships***<br>• *Able to visualise some objects from different perspective*<br>• *Able to use coordinates in first quadrant*<br>• *Beginning to manipulate visual images and coordinate information*<br><br>***Geometric Measurement***<br>• *Use one or two attributes (in sufficient) to explain their reasoning about measurement (e.g. consider length but forget impact of width/height)*<br>• *Beginning awareness of volume and capacity and the relationship between length, area and volume).* | **Consolidate and Establish**<br>➢ **2D** –<br>Experiences in different contexts where they explain their reasoning about shape identification (e.g. feely box)<br><br>Find/identify shapes presented in non-standard orientations with one or two specific properties<br><br>Construct specific shapes with compass and straight edge and/or Geogebra using properties<br><br>➢ **3D** – Drawing objects from different perspectives and building objects from different perspective drawings<br>➢ **Transformation and location** –<br>Identify 2D shapes that have been transformed under simple reflections and rotations<br><br>Identifying items on map from coordinates and placing items on map given coordinates for both street and Cartesian maps<br><br>➢ **Measurement –**<br>Order shapes and objects by area and volume justifying their choices<br><br>Recognise and identify specific angles such as right angle, straight angle and reflex angle.<br><br>**Introduce and Develop**<br>➢ Recognise parallel lines and right angle in situ<br>➢ Further develop geometric language such as diagonal, rotation perpendicular.<br>➢ Justify your answer – in groups so language is encouraged)<br>➢ Look at families of shapes/objects, describing what is the same and what is different<br>➢ Give directions on a map of their local area, using N S E & W and using perspective of traveller<br>➢ Introduce formal units of length and use them in calculation of perimeter, area and volume in open and/or real situations explaining solutions.<br>➢ Explore relationships between length, perimeter, area and volume |

| ZONE 4 Description | Teaching Implications |
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| *Know some geometric language, name some 3D objects and able to visualise objects from a different perspective but show incomplete reasoning in geometric and measurement situations, attending to necessary but not sufficient properties and using properties to identify shapes/objects. Perform measurement calculations but attend to only one attribute. Give directions from a map from personal rather than other viewer's perspective when situations are more complex.*<br><br>***Hierarchy and properties***<br>• *Recognise relevance of properties in more complex shapes and able to use some of the language but unable to use those properties or only focuses on one aspect.*<br>• *Recognise some conditions for a shape (e.g. square), but not dealing with all relevant information and cannot explain reasoning and unable to recognise minimum and sufficient conditions.*<br>• *Know names of some 3D objects (difference between prism and pyramids),*<br>• *Identify 3D objects from 2D formats such as isometric drawing and nets*<br>• *Generally incomplete reasoning in geometric situations*<br><br>***Transformation of Relationships***<br>• *Able to visualise objects from different perspective but limited capacity to explain (GPERS3.1).*<br>• *Give directions of a map from personal rather than other viewer's perspective*<br>• *Recognise same/different in transformation situations*<br><br>***Geometric Measurement***<br>• *Perform measurement calculations but attends to only one attribute,*<br>• *Recognise relevant aspects in informal measurement context, (e.g. GSOD 1.2) but in one aspect (linear measure) only.* | **Consolidate and Establish**<br>➢ **2D** –<br>Experiences identifying all properties of 2D shapes such as types of triangles and quadrilaterals.<br><br>Identify shapes from sets of properties (e.g. has 2 right angles and at least one pair of parallel lines)<br><br>Develop language such as diagonal and regular<br><br>➢ **3D** –<br>Investigate families of polyhedron and identify features that relate to the names (e.g. prisms and pyramids)<br><br>Use a variety of representation of 3D objects including nets, isometric and perspective drawings<br><br>Construct 3D objects from an isometric drawing or other representations.<br><br>Investigate drawings of joint 2D shapes to determine which ones create nets of simple 3D objects<br><br>➢ **Transformation and location** –<br>Reproduce 2D shapes that have been transformed under simple reflections and rotations<br><br>Give directions from one position to another on a map using different routes and using both perspective and directional approaches (e.g. go 2km turn right or go 2km south turn west).<br><br>➢ **Measurement –**<br>Explore volume and capacity using standard units<br><br>Given measurement situations with a range of information identify relevant information required to answer specific questions (e.g. GSODA1)<br><br>Experience ordering, estimating and measuring angles<br><br>Explore simple situations where angle sizes can be deduced (e.g. folding square diagonally to produce a 45° angle.<br><br>**Introduce and Develop** |

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| • *Incomplete reasoning in measurement situations* | ➢ Reason about geometric situations (e.g. do all triangles tessellate?)<br>➢ Describe all properties of a family of shapes/objects,<br>➢ Explore all possible nets of a simple object such as cube, triangular prism, square pyramid.<br>➢ Introduce and develop simple reflection and rotations on a coordinate grid<br>➢ Extend coordinate activities to 4 quadrants<br>➢ Apply measurement knowledge to solving real life situations (e.g. length of edging needed for garden bed, area of garden bed, etc). |

| ZONE 5 Description | Teaching Implications |
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| *Able to visualise and represent 3D objects using 2D platform (Nets); recognise properties in non-standard orientations and starting to use properties to identify classes; beginning to use but not recognise sufficient conditions; use either properties or orientations to reason in geometric situation; access relevant geometric language; demonstrate knowledge of dilation and coordinate systems and recognises some rotational symmetry; use landmarks but retain personal orientation when providing directions; provide partial solutions and explanations when calculating measurement situations. Begin to coordinate multiple components.*<br><br>***Hierarchy and properties***<br>• *Uses either properties or orientations to reason in geometric situation,*<br>• *Recognise parallel lines in non-standard representation, access to relevant language, able to recognise and use appropriate information to solve problem.*<br>• *Identify and recognise relevance of multiple representation 2D, beginning to use but not recognise sufficient condition, not recognising redundancy*<br>• *Use more complex language but cannot use to explain, can use in specific context*<br><br>***Transformation of Relationships***<br>• *Able to visualise and represent 3D objects using 2D platform (Nets),*<br>• *Demonstrate knowledge of dilation and coordinate systems,*<br>• *Uses landmarks but retains personal orientation when providing directions, have some insight into rotational symmetry* | **Consolidate and Establish**<br>➢ **2D** –<br>Explore similarities and differences between shapes; identifying those which are similar and those which are congruent<br><br>Extend the identification of 2D shapes using properties to include angle and diagonal properties, justifying their choices.<br><br>Explore classes of triangles and quadrilaterals, identifying properties.<br><br>Given one or two properties, identify all possible types of shapes<br><br>➢ **3D** –<br>Identify possible 3D objects from a group of properties (e.g. What shapes can you make if you have 2 equilateral triangles, 2 squares and 4 rectangles?; I have 3 pairs of parallel faces, and 6 faces, what am I?)<br><br>Create the nets of simple 3D objects<br><br>➢ **Transformation and location** –<br>Use knowledge of proportional reasoning to dilate shapes; calculating the new measurements from those of the original shape (some should include coordinates)<br><br>Explore the symmetry of shapes including bother reflectional and rotational symmetry<br><br>➢ **Measurement –**<br>Identify the relationships between length, perimeter and area.<br><br>Explore angles in polygons (including sum of interior angles)<br><br>Deduce angle size in situations involving parallel lines and transversal and in polygons.<br><br>Explore simple packaging situations using length and area<br><br>**Introduce and Develop**<br>➢ Construct own understanding of the hierarchy of quadrilaterals |

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| ***Geometric Measurement***<br>• *Provide partial solutions and explanations when calculating measurement situations.*<br>• *Use geometrical knowledge to find some angles but can't explain*<br>• *Coordinating multiple information but not necessary identifying /attending/recognising, what is not needed, how it is relevant to context, (e.g. not recognise box too big)* | ➢ Use geometric properties to argue in a variety of situations.<br>➢ Identify line of symmetry and rotational symmetry on a variety of shapes.<br>➢ Investigate volumes of simple cuboid and relate them to dimensions and surface area<br>➢ Reason about angle size in a variety of angle situations such as circles, polygons… |

| ZONE 6 Description | Teaching Implications |
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| *Use properties accurately when reasoning about spatial situations but lack knowledge of geometry hierarchy. Understand properties 2D shapes but not special cases (e.g. regular). Geometric and measurement arguments rely on examples/counter examples. Provide accurate directions from a map using appropriate language and able to describe directions from walker's perspective. Understand impact of doubling dimensions on volume, able to visualise volume and calculate when numbers are small. Omit one step when calculating multi-step measurement problems. Able to make deduction about angle situations with limited explanations. Beginning to reason deductively but not able to coordinate all aspects.*<br><br>***Hierarchy and properties***<br>• *Use properties accurately when reasoning about spatial situations but lack knowledge of geometry hierarchy*<br>• *Understand properties 2D shapes but not special cases (e.g. regular).*<br>• *Geometric arguments rely on examples/counter examples*<br><br>***Transformation of Relationships***<br>• *Recognise line symmetry but may over generalise and apply inappropriately.*<br>• *Provide accurate directions from a map, using appropriate language and able to describe directions from walker's perspective*<br><br>***Geometric Measurement***<br>• *Understand impact of doubling dimensions on volume, able to visualise volume and calculate when numbers are small.*<br>• *Able to make deduction about angle situations with limited explanations,*<br>• *Measurement arguments rely on examples/counter examples*<br>• *Omit one step when calculating multi-step measurement problems.*<br>• *Beginning to reason deductively.* | **Consolidate and Establish**<br>➢ **2D** –<br>Given the properties, identify the shapes<br><br>Explore the similarity and differences between classes of shapes<br><br>Given one or two properties, explore what else we know about the shape, describe all other properties (e.g. A shape has 4 equal side, what other properties must it have and what other properties might it have?)<br><br>➢ **3D** –<br>Identify classes of 3D objects from their properties<br><br>Investigate regular polyhedron (platonic solids)<br><br>➢ **Transformation and location** – Investigate mirror lines and points of rotation that are both within and outside shapes (including coordinate system)<br>➢ **Measurement –**<br>Reason about the sum interior angles of polygons<br><br>Explore measurement calculation problems which require multi-steps<br><br>Explore surface area and volume problems<br><br>**Introduce and Develop**<br>➢ Explore relationships between shapes starting to work towards a final hierarchical structure<br>➢ Use concept of regular, similar and congruent in argument<br>➢ Write definitions of shape and their properties in own words<br>➢ Explore isometric and non-isometric transformation of simple 2D shapes, particularly on a coordinate grid<br>➢ Apply measurement principles and calculations to complex shapes that require partitioning of shapes and explain/justify their solutions. |

| ZONE 7 Description | Teaching Implications |
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| *Work analytically with properties of rectangles. Beginning to recognise necessary and sufficient conditions. Use sound reasoning in argument/explanations, though explanations often are procedurally based or base on an example. Able to recognise the relationship between length, area and volume. Using multiple properties to reason but in measurement situations may rely on procedural explanations.*<br><br>***Hierarchy and properties***<br>• *Can recognise necessary and sufficient conditions for all triangle families of shapes*<br>• *Working analytically with properties of rectangles, recognising necessary and sufficient conditions for rectangle and square*<br>• *Recognise and use concepts of similarity and congruence in argument.*<br>• *Beginning to recognise necessary and sufficient conditions,*<br>• *use sound reasoning in argument/explanations*<br><br>***Transformation of Relationships***<br>• *Can apply isometric transformations and simple dilations to 2D shapes using a variety of platform including the coordinate systems*<br><br>***Geometric Measurement***<br>• *Explanations often are procedurally based*<br>• *Able to recognise the relationship between length, area and volume*<br>• *Able to calculate area of shapes on a coordinate system* | **Consolidate and Establish**<br>➢ **2D** –<br>Identify necessary and sufficient condition (e.g. Given a shape has 4 equal sides, what other property is needed to ensure that it is a square?)<br><br>Proving the properties of shapes based on concept of similarity and congruence (e.g. Given that a quadrilateral's diagonals bisect the angles, prove that it is a rhombus).<br><br>➢ **3D** – Investigate cross sections of 3D objects (e.g. identifying faces of cross section cuts).<br>➢ **Transformation and location** – Explore the effect of a transformation on a shape in a coordinate system<br>➢ **Measurement –**<br>Experience finding area of polygonal and circle shapes presented in non-prototype orientation (e.g. shapes presented on coordinate grid).<br><br>Identify the relationships between surface area and volume<br><br>Able to use knowledge of 2D shapes, similarity and congruent to determine the size of angles and make justification<br><br>**Introduce and Develop**<br>➢ Using principles of geometric properties to reason and justify conjectures/propositions (e.g. why are there only 5 platonic solid, which regular shapes can tessellation, justify your reasoning, etc)<br>➢ Construct a variety of shapes under particular reflectional and rotational conditions (e.g. draw a shape that has exactly two lines of symmetry and a rotational symmetry of 2; construct a shape that has rotational symmetry of 3 and no line of symmetry)<br>➢ Use measurement in a variety of 2D/3D situations including angles in a circle, Pythagoras and trigonometry to solve problems |

| ZONE 8 Description | Teaching Implications |
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| *Construct arguments based on multiple properties of 2D shapes and 3D objects, using the necessary and sufficient conditions to reason about geometric and measurement situations, conjectures and propositions (theorem), demonstrate understanding of both reflectional and rotational symmetry.*<br><br>***Hierarchy and properties***<br>• *Can recognise necessary and sufficient conditions for all quadrilateral families of shapes and polygons and circles*<br>• *Constructs arguments based on multiple properties of 2D shapes and 3D objects,*<br>• *Uses the necessary and sufficient conditions to reason about geometric situations, conjectures and propositions (theorem)*<br><br>***Transformation of Relationships***<br>• *Demonstrate understanding of both reflectional and rotational symmetry.*<br><br>***Geometric Measurement***<br>• *Uses concept in measurement clearly to argue about conjectures, propositions and real situations* | **Consolidate and Establish**<br>➢ **2D** –<br>Create their own diagram showing the hierarchy of quadrilaterals.<br><br>Construct different types of quadrilaterals and other polygons (e.g. using platforms such as compass and ruler construction and geogebra).<br><br>Make conjectures about properties of shapes and justify/prove their conjectures (e.g. join the midpoints of a rectangle, make a conjecture about the shape formed and prove it)<br><br>➢ **3D** – Develop arguments about 3D objects (e.g. agree or disagree with the statement… giving justification for your position [There can only be 5 regular polyhedral])<br>➢ **Transformation and location** – Explore the effect of sequential transformations on a shape in a coordinate system<br>➢ **Measurement –** Use measurement in problem situations, justifying solution<br><br>**Introduce and Develop**<br>➢ Discuss what constitutes proof and explore its use in a variety of geometric and measurement situations<br>➢ Extend hierarchy ideas to 3D<br>➢ Extend measurement to more complex 2D/3D situations<br>➢ Apply measurement in real problem contexts |