Name: ……..

Year Level: …… Date: …….

ALGEBRA FORM D

Assessment Booklet

Reframing Mathematical Futures II

An Australian Mathematics & Science Partnership Project (2015-2018)

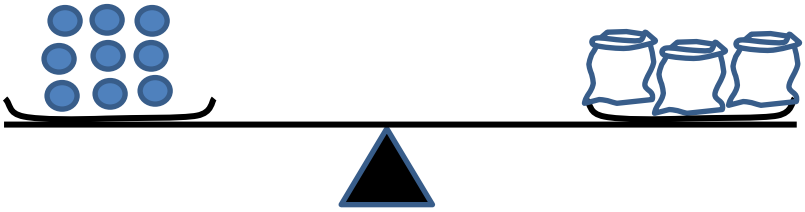
Developed by the RMFII Research Team - RMIT University, June 2019

ALGEBRA FORM D

1 Balancing Scales

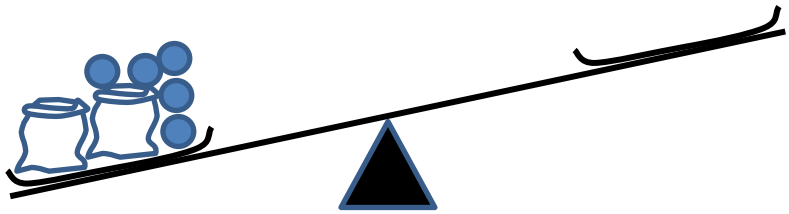
The bags on these scales all weigh the same.

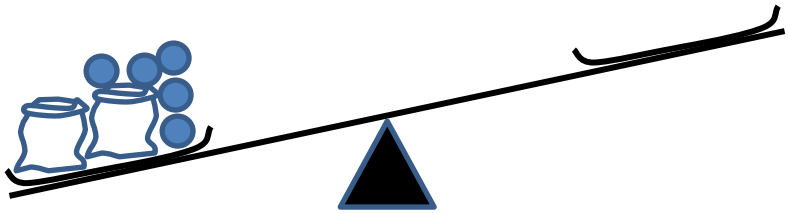
The balls on these scales all weigh the same.



a [AEQB1]

Using the information above, draw two different ways to make the scales balance (you may use as many bags and/or balls as you wish).





b [AEQB2]

Write an equation for each scale you drew.

_____

_____

2 Board Room Tables

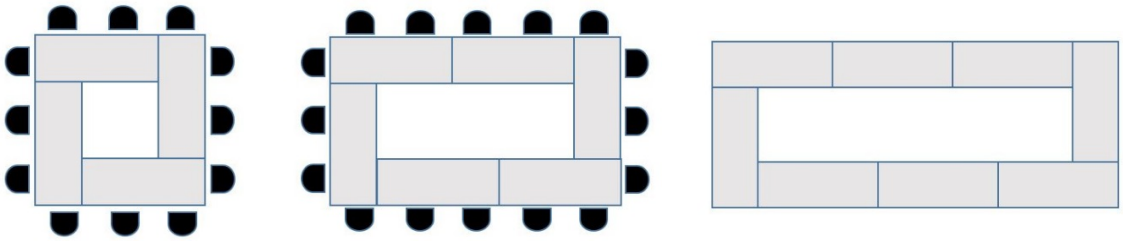
In order to be flexible a Board Room has several tables that can be arranged to cater for different numbers of people at Board meetings.

Each table is a rectangle.

Each table can seat one person on its short edge and two people on its long edge.

The diagrams below show how these tables can be arranged for different numbers of people.

(No one sits inside the arrangement.)



Size 1	Size 2	Size 3
4 tables	6 tables	
12 people	16 people	

a [ABRT2]

How many tables are in a Size 4 arrangement? _____

b [ABRT3]

How many tables are in a size 7 arrangement? _____

Explain your reasoning.

c [ABRT4]

Write down in words or symbols a rule for working out the number of tables when you know the size number.

3 **Number Sentences**

[ARELS7]

What can you say about the relationship between c and d in this equation?

$$c \times 2 = d \times 14$$

4 [AEQEX5]

Marika claimed that $6x + 3 - 2x$ is the same as $\frac{(8x + 6)}{2}$

Do you agree or disagree with Marika's claim?

Explain your reasoning

5 [ASAND]

Sandra added different sequences of odd numbers starting at 1. That is, 1, 1 + 3, 1 + 3 + 5 and so on.

She noticed a pattern in her results.

Describe the pattern and explain your reasoning.

6 Is it hot enough?

Cooking shows have become very popular around the world.

One problem is that American recipes use different systems of measurement to those used in Australia. Americans measure temperature in Fahrenheit (*F*) and Australians measure temperature in Celsius (C).

The relationship is $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$

a [AHOT1]

What is a temperature of 20° Celsius in Fahrenheit?

b [AHOT2]

What would a temperature of 300° Fahrenheit be in Celsius?

c [AHOT2a]

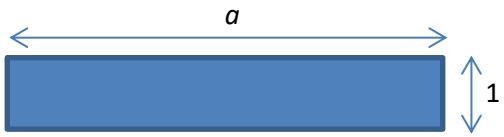
Sam wants a rule for finding the temperature in Celsius given the temperature in Fahrenheit.

Describe a rule in words or symbols that Sam might use.

7 Tile Pieces

a [ATILP1]

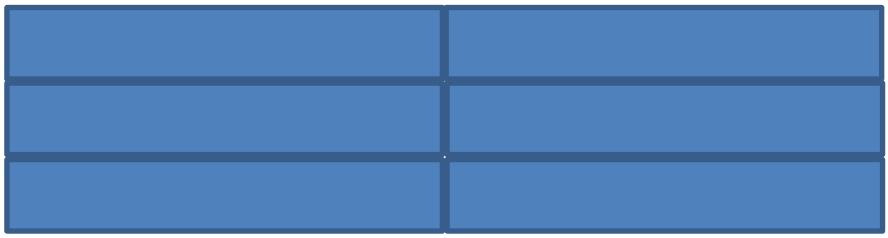
This tile has length of a units and width of 1 unit.



What is the perimeter of the tile (without using a ruler)?

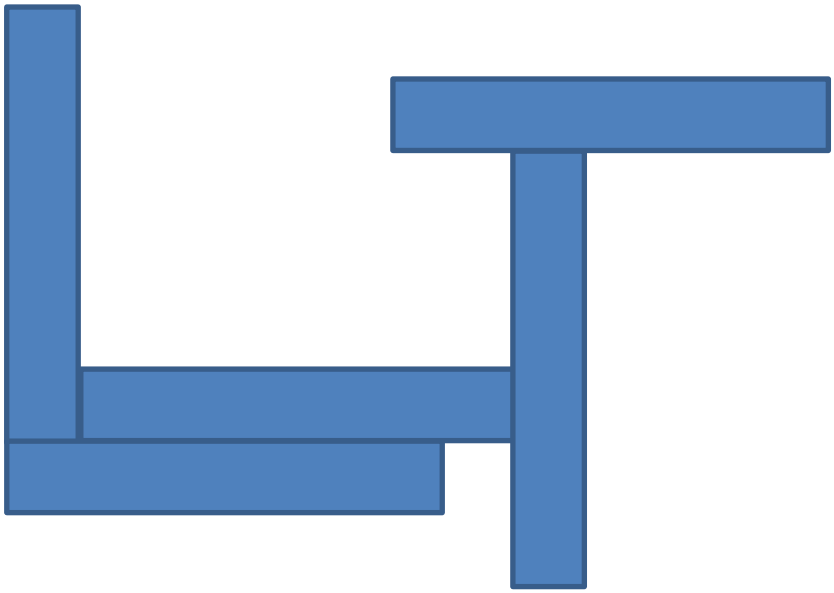
b [ATILP2]

The shape below is made of the same tiles. Without using a ruler, what is the perimeter of the tile?



c [ATILP4]

The shape below is made of the same tiles. Again, without using a ruler, what is the perimeter of this shape? Explain your reasoning.



ALGEBRA FORM D RUBRIC

1. AEQB1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Attempt made but uses incorrect weights of bags and balls (i.e., does not use 1 bag = 3 balls)
2	One way drawn correctly may replicate left hand side (i.e., 2 bags and 5 balls)
3	Two different ways drawn correctly (e.g., 0 bags, 11 balls; 1 bag, 8 balls or 3 bags, 2 balls)

AEQB2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	One equation written correctly or expression only written (e.g., 3x + 2)
2	Two equations written correctly (e.g., 2 bags + 5 balls = 3 bags + 2 balls or 2x + 5 = 3x + 2)

2. ABRT2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (10 or 10 tables)

ABRT3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (16 tables) with no explanation or evidence of additive thinking (e.g., continues table or *it goes up by two* or *add two each time*))
2	Correct response with reasonable explanation either in words (e.g., *The number of tables along each long side of the arrangement is the same as the Size number, so you multiply this by two and add the two tables, one for each end*) or in symbols (N = 2S + 2 or 2(S + 1)).

ABRT4

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Response suggests additive thinking (e.g., *goes up by two* or *add two each time*)
2	Correct with evidence of multiplicative thinking expressed in words (e.g., *two times the number of tables plus two*) or in symbols N = 2 x size + 2

3. ARELS7

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Specific solution provided (e.g., *c must be 7 and d must be 1 to make it a true number sentence)* or a general statement *(e.g., c is bigger than d*)
2	Starement correctly describes relationship (e.g., *c is 7 times the number d)*

4. AEQEX5

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Disagrees with Marika's claim, may make an error in simplifying expression or not realise that doubling and halving leaves the initial expression unchanged OR Agrees with little/no reasoning
2	Agrees with Marika's claim with clear explanation that recognises $6x + 3 – 2x = 4x + 3$ and that doubling and dividing by two leave the expression unchanged (e.g., if you doubled that it would be equal to $8x + 6$, dividing it by 2 just brings it back to $4x + 3$)

5. ASAND

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Lists successive sums (for combinations given (e.g., 1, 4, 9), may include next term in pattern (i.e., 1 + 3 + 5 + 7 = 16) and/or recognise these as square numbers (e.g., *it looks like square numbers*) but does not test the pattern beyond 16.
2	Identifies pattern correctly as the sequence of square numbers and provides at least one example beyond 16
3	Describes the general pattern correctly (e.g., the sum of the first n odd numbers is equal to n^2)

6. AHOT1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect response but attempt to use formula, may make an error in calculation
2	Correct response (68°F or 68)

AHOT2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect response but attempt to use formula, may make an error in transposition and/or calculation
2	Correct response (148.8°C or 149°C or 149)

AHOT2A

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but some attempt to transpose formula (e.g., recognises multiplication and/or division is involved but unable to specify correctly, may or may not recognise subtraction)
2	Partially correct but makes and error in transposing or omits a step in either words (e.g., *take 32 from F temperature then multiply by 5*) or symbols (e.g., F – 32 = 9C or C = 5F/9 -32))
3	Correct rule described in either words (e.g*., take 32 from the temperature in Fahrenheit, multiply the result by 5, then divide by 9*) or in symbols (C = 5(F-32)/9)

7. ATILP1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Calculation based on numbers (e.g., assumes a = 5 or 6 cm) that shows an understanding of perimeter (e.g., 5 + 1 + 5 + 1 = 11 or 12 + 2 = 14)
2	Correct response but not in its simplest form (e.g., a + a + 1 + 1)
3	Correct, simplified response (2a + 2 or 2(a + 1))

ATILP2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect, but attempt made to solve for the perimeter using symbols (e.g. a + a + 3 + 3; 4 a + 3) or assumes a = 5 or 6 cm (e.g., 10 + 3 + 10 + 3 or 12 + 12 + 3 + 3)
2	Correct response but not in its simplest form (e.g., 2a + 2a + 3 + 3)
3	Correct symbolic response in simplified form (e.g., 4a + 6 or 2(2a + 3))

ATILP4

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but partly identifiable response (e.g., 5a + 7 based on counting up recognisable a's and 1s), with little/no working or explanation to support response OR incorrect response based on a = 5 or 6 cm
2	Incorrect response due to minor errors OR correct response based on a = 5 or 6 cm with working that shows an understanding of perimeter
3	Correct symbolic response but not simplified (e.g., a + 1 + 1 + a - 1 + a + a - 1 + a – 1 + 1 + a + 1 + a + 1 + 1 + 1 + a) and with little/no reasoning to support conclusion
4	Correct symbolic response (8a + 4) without clear explanation (e.g. just added all sides together) OR correct response but not simplified with a reasonable explanation for solution (e.g., see below)
5	Correct response (8a + 4) with clear explanation (e.g., sides clearly labelled and partially covered sides such as the bottom of the rectangle forming the 'T' are explained in terms of a – 1) OR explanation based on visualisation such as the horizontal side lengths of the rectangle forming the 't' shape are 2a since the unit at the base of the 'T' is equal to the covered piece of top

STUDENT SCORE SHEET ALGEBRA FORM D

Student Name:	Year Level:

		Score	Comments
1	AEQB1		
	AEQB2		
2	ABRT2		
	ABRT3		
	ABRT4		
3	ARELS7		
4	AEQEX5		
5	ASAND		
6	AHOT1		
	AHOT2		
	AHOT2a		
7	ATILP1		
	ATILP2		
	ATILP4		
Total Raw Score			

RAW SCORE TRANSLATOR FOR ALGEBRA FORM D

The following table locates students on the **Learning Progression for Algebraic Reasoning** based on their total score for Algebra Form D. Total scores are obtained by adding the rubric scores assigned to each item on the form. Where a total score is very close to the beginning or end of a score range, teachers are advised to use their knowledge of the student to make a decision about the most appropriate Zone.

Students need to have had an opportunity to attempt all tasks for this process to be meaningful.

Total Score	Zone	Zone Description
32-35	8	Is able to combine a facility with symbolic representation and an understanding of algebraic concepts to represent and explain mathematical situations. Explanations are sophisticated using logical thought and the language of reasoning. Can use multiple representations in a co-ordinated manner to solve, analyse, convince and conclude. Can visualise the form and structure of a function, at least graphically, from a real context. Is able to work in a context free environment using symbolic language and treat algebraic expressions (e.g. 3x + 2) as single entities. Can generalise more complex situations. Is able to establish and describe equivalence involving the four operations explaining relationships in symbolic terms. Can use abstract symbols to solve problems in context with multiple steps.
28-31	7	Is able to use and interpret algebraic conventions for representing generality and relationships between variables. Beginning to use sound logical reasoning with appropriate reasoning language (e.g. if … then, must) evident. There is more co-ordination of multiplicative thinking and the associated language to notice algebraic structure. Can recognise and use the relationships between multiple entities and connections between and within different representations. Is able to establish and describe equivalence explaining relationships using the distributive property and the inverses of addition and multiplication. Can generalise quite complex situations and in more direct situations is beginning to use simplest form.
20-27	6	Can use and interpret basic algebraic conventions to represent situations involving a variable quantity. Beginning to explain using logical language and to use if … then reasoning. Uses symbolic language but the need for simplification is still being developed. Able to generalise arithmetic relationships with justification, including simple multiplicative relationships, but are often still context bound. Can show why several expressions are equivalent, typically employing numerical (non-symbolic) justifications.
13-18	5	Able to use multiplicative reasoning in simple situations. Can reason with more complex additive situations involving larger numbers and subtraction but usually by examples. Has moved from algebraic expressions to using equations. Can derive a strategy that maintains equivalence, but cannot

		yet generalise the situation. Able to use symbols to express rules. Can follow, compare and explain rules for linking successive terms in a sequence. Recognises and represents simple functional representations. Can justify an argument using mathematical text. Beginning to generalise using words or using some symbolic generalisations in simple situations, usually building on in context.
11-12	4	Beginning to work multiplicatively and simultaneously co-ordinate variables, although still uses specific examples to convince. Able to reason and generalise in simple situations. Can recognise and interpret the relevance of range from table and/or graphs and to recognise functional relationships. When faced with more complex algebraic situations is unable to use the full range of explanation or handle all of the information simultaneously. Beginning to transition to abstraction by inserting a number for a pronumeral.
6-10	3	Beginning to use symbolic expression and elementary reasoning. While still using arithmetic approaches there is evidence of relational reasoning with the numbers and providing some explanation. Beginning to recognise simple multiplicative relationships. There is some evidence of co-ordination of two ideas. Explanation and justification is limited. Algebraic expressions are used rather than equations. Beginning to recognise equivalent relationships. Can explain simple generalisations by telling stories, manipulating materials and very simple use of symbolic language.
4-5	2	Beginning to recognise patterns and relationships and conjecture about these. Able to identify numbers that vary and numbers that stay the same. Engages with the context, but arithmetic reasoning, typically based on calculations, is still being used. Recognises some multiples and some relationships like 6 more/6 less, while not necessarily recognising equivalence. Can work with simple scales and transfer from a table of values to a graph.
1-3	1	Can continue simple patterns but is likely to build them additively. Reasoning is confined to specific incidences and numerical examples of simple physical situations. Arithmetic thinking is used. Abstraction and generalisation not evident at this stage.