Name: ……..

Year Level: …… Date: …….

ALGEBRA FORM C

Assessment Booklet

Reframing Mathematical Futures II

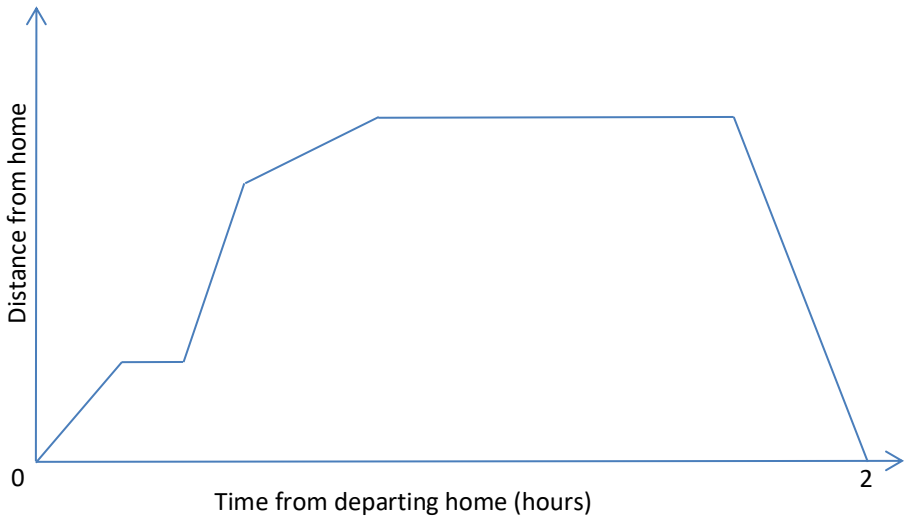
An Australian Mathematics & Science Partnership Project (2015-2018)

Developed by the RMFII Research Team - RMIT University, June 2019

ALGEBRA FORM C

1. [ACART1]

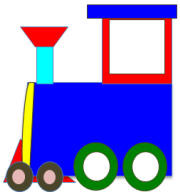
The graph below represents a car trip to the local supermarket. Write a short story explaining what the graph tells you about the trip.



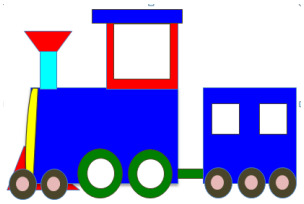
2. [AMUST]

Mustafa said "the sum of two odd numbers is always an even number". Do you agree? Show your reasoning to convince someone else that your decision is correct.

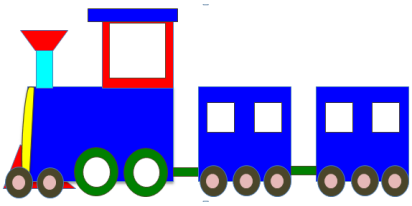
3 Trains



toy train size 1



toy train size 2



toy train size 3

The engine of the train has 8 wheels, 4 on each side, and each carriage has 6 wheels, 3 on each side.

The table shows the number of wheels on each train:

Train size	1	2	3	4	5	6
Number of wheels	8	14				

a [ATRNS1]

Fill in the table to show the number of wheels for the trains size 3, 4, 5 and 6.

b [ATRNS2]

The largest train set in the toy shop is size 15.

How many wheels does the size 15 have? _____

Explain your reasoning using as much mathematics as you can.

c [ATRNS3]

Ben says his train has exactly 60 wheels. Can Ben be correct? _____

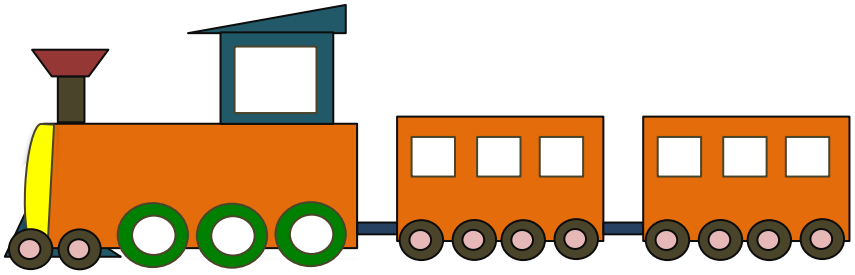
Explain your reasoning using as much mathematics as you can.

d [ATRNS4]

Write down in words or symbols a rule for working out how many wheels any sized train would need.

e [ATRNS5]

The toy shop decides to introduce a new Super Train set. In the new train set the engine has 10 wheels and each carriage has 8 wheels.



Super Train Size 3

How many wheels does a Super Train Size 7 have?

Show your reasoning.

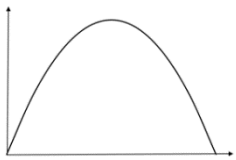
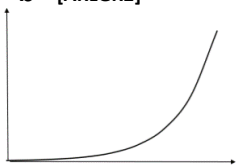
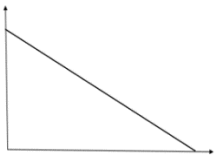
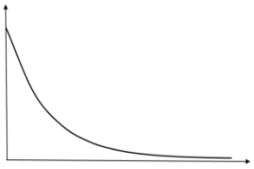
f [ATRNS6]

Write a rule in words or symbols for working out the size Super Train given any number of wheels.

4. Real Graphs

Graphs are used to represent real situations. A number of real situations are described below. Write the letter of the situation next to the graph which represents it and give the reason for your choice. Not all situations have a matching graph.

A	A candle is lit. The graphs show the height of the candle against the time it has been burning.	**B**	A number of bottles of cool drink are purchased. The graph shows the number of bottles purchased against the total cost.
C	A piece of paper is folded in half to form two parts. The result is then folded in half repeatedly and after each fold the number of parts is counted. The graph shows the number of folds against the number of parts made.	**D**	The rate of radioactive decay uses a measure called half-life, which is the time it takes for half of the atoms of a radioactive substance to disintegrate. The graph shows the amount of radioactive material left against time measured over several half-lives.
E	A javelin is thrown. The graph shows the height of the javelin against the time it is in the air.	**F**	A diver dives from a 10 metre platform. The graph shows the height of the diver above the water against the time from the start of the dive.

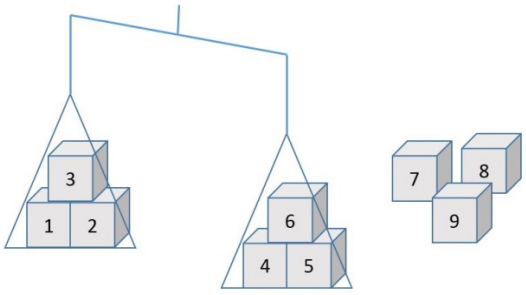
Graph	Situation	Reason
a **[ARLGR1]** 		
b **[ARLGR2]** 		
c **[ARLGR3]** 		
d **[ARLGR4]** 		

5. Boxes

There are nine boxes that all look exactly the same, but one is a bit heavier than the others.

Carla says “I can use the scales to find the heavy one in just two steps”.

This is what Carla does first.



a [ABOX1]

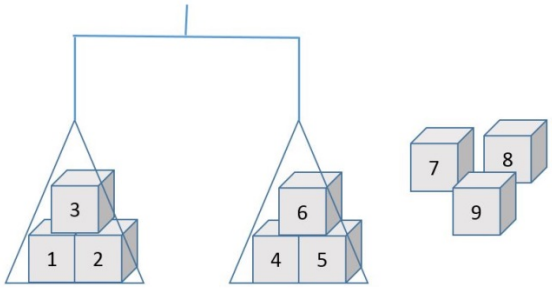
Explain what Carla now knows about the heavy box.

b [ABOX2]

What should Carla do next? Explain your reasoning.

c [ABOX3]

Suppose the scales showed this the first time instead. What should Carla do to find the heavy box?



ALGEBRA FORM C RUBRIC

1. ACART1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Story explains 1 to 3 elements of the graph
2	Story partially correct (explains at least 4 elements in the graph)
3	Plausible story that makes sense of all elements in the graph.

2. AMUST

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct (Yes) with little or no reasoning or examples to support conclusion
2	Correct supported by at least one example
3	Correct supported by generalised argument (e.g., Odd + Odd = Even + 1 + Even + 1 = Even + 2 = Even number).
4	Correct supported by a symbolic argument (e.g., recognses even numbers 2n and odd numbers as 2n ± 1)

3. ATRNS1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	At least two entries correct
2	Table completed correctly (20,26,32,38)

ATRNS2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (92) with no explanation/working or incorrect response with working to show some understanding of pattern or incorrect with working to show minor calculation error
2	Correct response with an explanation that reflects the use of an additive strategy (e.g., *goes up by 6* or continues table to a train size of 15)
3	Correct response with an explanation of a multiplicative approach expressed in words or as a rule but not in simplest form (e.g., *you multiply 6 by 14 and add 8 or 6 x 14 + 8*)
4	Correct response with an explanation of a multiplicative approach expressed in words or as a rule in simplest form that recognises the 6 wheels in the engine (e.g., *you need to times 15 by 6 and add 2* or 15 x 6 + 2)

ATRNS3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (No) but with no explanation OR incorrect response with sound reasoning
2	Correct response with reasoning to support conclusion (e.g., *60 – 8 is 52 and 52 is not divisible by 6* or *a size 9 train would have 56 wheels and a size 10 train would have 62 wheels so you can't have a train with 60 wheels*.

ATRNS4

SCORE	DESCRIPTION
0	No response or irrelevant response
1	General statement (e.g., *it goes up by 6*) OR incorrect but some evidence that multiplication involved, may or may not recognise addition
2	Rule correctly expressed in words (e.g., *you multiply 6 by one less than the train Size number and you add 8*) or in symbols but not in simplest form (e.g., N = 8 + 6S – 6 or N = 8 + 6 (S – 1))
3	Rule correctly expressed in words or symbols in simplest form that recognises the 8 wheels in the engine (e.g., *you need to times S by 6 and add 2 or N = 6S + 2*)

ATRNS5

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct response (58) with little/no explanation/working OR incorrect response with working to show some understanding of pattern, may involve a minor calculation error
2	Correct response with an explanation that suggests the use of an additive strategy (e.g., *goes up by 8* or uses a table for Super Train Sizes from 1 to 7)
3	Correct response with an explanation that indicates a multiplicative approach expressed either in words (e.g., *you multiply 8 by one less than the Size and you add 10)* OR symbols (e.g., 10 + 6 x 8 or 2 + 7 x 8)

ATRNS6

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Incorrect but some evidence of multiplicative thinking (e.g., recognises division is involved but unable to specify correctly, may or may not recognise subtraction)
2	Correct rule with reasonable explanation either in words (e.g., *you take 2 from the number of wheels and divide by 8*) or in symbols (e.g., S = (N – 2)/8)

4. ARLGR1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct situation (E) identified with little/no reasoning OR incorrect situation with plausible reasoning that ignores one or more aspects (e.g., *Situation F, diver jumps up from the platform before diving into the water, so the graph should go up then down* (did not take into account that height would not start at 0))
2	Correct situation (E) with an appropriate explanation related to the graph (e.g., Situation E – the javelin will start low then increase its height from the ground before dropping back down, so the graph should go up and then down)

ARLGR2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct situation (C) identified with little/no reasoning OR incorrect situation with plausible reasoning that ignores one or more aspects (e.g., *Situation B – The more bottles you purchase, the greater the cost so the graph should be increasing* (did not take into account rate))
2	Correct response (C) with an appropriate explanation related to the graph (e.g., *Situation C – With each fold of the paper, the number of parts doubles so the graph should increase at a faster rate* (exponentially)).

ARLGR3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct situation (A) identified with little/no reasoning OR incorrect situation with plausible reasoning that ignores one or more aspects (e.g., *Situation D – Radioactive decay is the disintegration of radioactive material so the graph should be decreasing* (did not take into account half-life))
2	Correct response (A) with an appropriate explanation related to the graph (e.g., *Situation A – the candle will steadily decrease in height as it burns out so the graph should be decreasing as a steady rate*).

ARLGR4

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Correct situation (D) identified with little/no reasoning OR incorrect situation with plausible reasoning that ignores one or more aspects (e.g., *Situation A – the candle decreases in height as it burns out but the wick may still burn even if the wax is gone, so graph should decrease more slowly at the end* (did not take into account rate))
2	Correct response (D) with an appropriate explanation related to the graph (e.g., *Situation D –Half of the atoms disintegrate with each halflife, so the graph should decrease at a faster rate*).

5. ABOX1

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Partially correct response – the heavy box could be either 4, 5, 6, 7, 8 or 9 (i.e., does not realise 7, 8 and 9 could be ruled out)
2	Correct response – the heavy box could be either 4, 5 or 6 OR the heavy box is on the right hand side of the scale.

ABOX2

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Selects less efficient methods (e.g., weigh boxes 4, 5, 6, 7, 8 and 9 OR weigh all boxes one at a time).
2	Incomplete explanation (e.g., need to weigh boxes 4, 5 and 6 OR weigh 4 and 5 with little/no further reasoning).
3	Correct response, that is, weigh any two of boxes 4, 5 or 6 (e.g., need to weigh boxes 4 and 5 leaving box 6 out. If 4 and 5 are the same, box 6 is the heavy box. If scale is unbalanced, the heavy box is in the lower scale pan).

ABOX3

SCORE	DESCRIPTION
0	No response or irrelevant response
1	Selects less efficient methods (e.g., compare weight of boxes one by one OR weigh boxes 7, 8 and 9 one at a time against box 4).
2	Incomplete explanation (e.g., weigh boxes 7, 8 and 9 OR weigh 7 and 8 with little/no further reasoning).
3	Correct response, that is, weigh any two of boxes 7, 8 or 9 (e.g., weigh boxes 7 and 8 leaving box 9 out. If 7 and 8 are the same, box 9 is the heavy box. If scale is unbalanced, the heavy box is in the lower scale pan).

STUDENT SCORE SHEET ALGEBRA FORM C

Student Name:	Year Level:

		Score	Comments
1	ACART1		
2	AMUST		
3	ATRNS1		
	ATRNS2		
	ATRNS3		
	ATRNS4		
	ATRNS5		
	ATRNS6		
4	ARLGR1		
	ARLGR2		
	ARLGR3		
	ARLGR4		
5	ABOX1		
	ABOX2		
	ABOX3		
Total Raw Score			

RAW SCORE TRANSLATOR FOR ALGEBRA FORM C

The following table locates students on the **Learning Progression for Algebraic Reasoning** based on their total score for Algebra Form C. Total scores are obtained by adding the rubric scores assigned to each item on the form. Where a total score is very close to the beginning or end of a score range, teachers are advised to use their knowledge of the student to make a decision about the most appropriate Zone.

Students need to have had an opportunity to attempt all tasks for this process to be meaningful.

Total Score	Zone	Zone Description
37-39	8	Is able to combine a facility with symbolic representation and an understanding of algebraic concepts to represent and explain mathematical situations. Explanations are sophisticated using logical thought and the language of reasoning. Can use multiple representations in a co-ordinated manner to solve, analyse, convince and conclude. Can visualise the form and structure of a function, at least graphically, from a real context. Is able to work in a context free environment using symbolic language and treat algebraic expressions (e.g. 3x + 2) as single entities. Can generalise more complex situations. Is able to establish and describe equivalence involving the four operations explaining relationships in symbolic terms. Can use abstract symbols to solve problems in context with multiple steps.
31-36	7	Is able to use and interpret algebraic conventions for representing generality and relationships between variables. Beginning to use sound logical reasoning with appropriate reasoning language (e.g. if … then, must) evident. There is more co-ordination of multiplicative thinking and the associated language to notice algebraic structure. Can recognise and use the relationships between multiple entities and connections between and within different representations. Is able to establish and describe equivalence explaining relationships using the distributive property and the inverses of addition and multiplication. Can generalise quite complex situations and in more direct situations is beginning to use simplest form.
28-30	6	Can use and interpret basic algebraic conventions to represent situations involving a variable quantity. Beginning to explain using logical language and to use if … then reasoning. Uses symbolic language but the need for simplification is still being developed. Able to generalise arithmetic relationships with justification, including simple multiplicative relationships, but are often still context bound. Can show why several expressions are equivalent, typically employing numerical (non-symbolic) justifications.
20-27	5	Able to use multiplicative reasoning in simple situations. Can reason with more complex additive situations involving larger numbers and subtraction but usually by examples. Has moved from algebraic expressions to using equations. Can derive a strategy that maintains equivalence, but cannot

		yet generalise the situation. Able to use symbols to express rules. Can follow, compare and explain rules for linking successive terms in a sequence. Recognises and represents simple functional representations. Can justify an argument using mathematical text. Beginning to generalise using words or using some symbolic generalisations in simple situations, usually building on in context.
15-19	4	Beginning to work multiplicatively and simultaneously co-ordinate variables, although still uses specific examples to convince. Able to reason and generalise in simple situations. Can recognise and interpret the relevance of range from table and/or graphs and to recognise functional relationships. When faced with more complex algebraic situations is unable to use the full range of explanation or handle all of the information simultaneously. Beginning to transition to abstraction by inserting a number for a pronumeral.
11-14	3	Beginning to use symbolic expression and elementary reasoning. While still using arithmetic approaches there is evidence of relational reasoning with the numbers and providing some explanation. Beginning to recognise simple multiplicative relationships. There is some evidence of co-ordination of two ideas. Explanation and justification is limited. Algebraic expressions are used rather than equations. Beginning to recognise equivalent relationships. Can explain simple generalisations by telling stories, manipulating materials and very simple use of symbolic language.
5-10	2	Beginning to recognise patterns and relationships and conjecture about these. Able to identify numbers that vary and numbers that stay the same. Engages with the context, but arithmetic reasoning, typically based on calculations, is still being used. Recognises some multiples and some relationships like 6 more/6 less, while not necessarily recognising equivalence. Can work with simple scales and transfer from a table of values to a graph.
1-4	1	Can continue simple patterns but is likely to build them additively. Reasoning is confined to specific incidences and numerical examples of simple physical situations. Arithmetic thinking is used. Abstraction and generalisation not evident at this stage.