Zone 8 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 8.

reSolve

Algebra: Tens and Ones (Strategies for simplifying expressions)
Mathematical Modelling: Introduction (Traffic Jams and Queues) (Broader range of multiplicative situations)
Mathematical Modelling: Packaging Designer (Broader range of multiplicative situations)
Mathematical Modelling: Pricing for Profit (Broader range of multiplicative situations)

Maths300

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Ice Cream Flavours: Zones 4 – 8
This task investigates the number of ice-creams that can be made given any number of flavours, any number of scoops and whether or not repeats are allowed.

Cracked Tiles: Zones 5 – 8
This task is one of several where geometric patterns lead to algebraic investigations and generalisations. This particular task uses rectangular arrays of tiles when investigating how many tiles need to be replaced when an electrician lays a cable along the diagonal of the rectangle.

Division Boxes: Zones 5 – 8
This task uses divisibility tests to develop a strategy for solving a divisibility problem. It quickly becomes an open-ended investigation. There is software to support the investigation.

Eric the Sheep: Zones 5 – 8
This task involves students in identifying a surprising pattern that can be linked to the physical structure of the problem. The pattern can be described in words or algebraically and can be extended to include step functions, domain and range.

Factors: Zones 5 – 8
There is a rule which can tell you how many factors exist for any number. This investigation is designed to both uncover that rule and see the logic behind it. The investigation helps develop a holistic understanding of how factors and prime factors are interconnected.

The Mushroom Hunt: Zones 5 – 8
This task can be tackled at every level from an exploration of doubling, to an introduction to powers and indices, to the concept of binary numbers, to an investigation of multiplicative (exponential-like) growth. At each level application of problem solving strategies is required.

Garden Beds: Zones 6 – 8
This task has a very rich context from which many mathematical concepts can be explored. The mathematics of counting, area and perimeter, pattern and algebra ideas are all very evident. Seeing the construction of the garden in different ways leads into explaining patterns using algebra rules, equivalence of algebraic expressions, expanding brackets and collecting terms.

Heads and Legs: Zones 6 – 8
In this task students are asked to find how many of each type of animal there are given the number of heads and the number of legs they have collectively. The task can be solved in a variety of ways from drawing animals, or using the accompanying software to solving simultaneous equations.

Algebra Charts: Zones 7 – 8
This task builds on the Number Charts task and involves algebraic simplification and using the distributive property to factorise and expand algebraic expressions. It seamlessly links the inverse operations.

Baby in the Car: Zones 7 – 8
This task uses mathematical modelling to investigate the surface area to volume ratio.

Country Maps: Zones 7 – 8
This task investigates area and ratios in the context of comparing the areas of the states of Australia to each other.

Staircases: Zones 6 – 8
This task uses the visual pattern of the steps in a staircase. The discovery of the pattern opens the door to further algebra and to a visual representation of the pattern in graphical form, as well as the generalisation of the triangular numbers.

Twelve Days of Christmas: Zones 7 – 8
This task investigates the number of presents from the popular Twelve Days of Christmas song. It includes triangular numbers, square numbers and extends into quadratic functions.

Biggest Volume: Zone 8 –
This task presents an investigation involving maximising the volume of a cuboid. It involves decimal fractions and gives students some pre-calculus notions such as limits.

Cylinder Volumes: Zone 8 –
This task presents an investigation to compare the volumes of cylinders that have the same surface area and how to maximise the volume. There is supporting software.

Natalie's LCM Task: Zone 8 –
This task tweaks a typical text book question on lowest common multiples to become a rich investigation of finding possible numbers when given the LCM. Prime factors are utilised and the generalisations for different groups of factors are investigated.

Newspaper Cubes and Volume of a Room: Zone 8
This lesson is about visualising volumes of cubes and cuboids. Additional benefits are experience of structural engineering features such as the need to use triangles to make objects rigid and consequent connections with students' real life experiences of building. The lesson makes the calculation of volume a richer experience than merely multiplying three numbers.

Pyramid Puzzle: Zone 8 –
The starting point of this task is a 3D spatial puzzle with only four pieces. The challenge is to make a tetrahedron. Further problem solving begins by asking the students to predict the number of spheres in the next level of the tetrahedron. For some the development and justification of this prediction may be sufficient learning for now, but the problem offers much more. There is a much mathematics which includes summing square, cube and triangle numbers; the concept of proof; the volume of a pyramid; links with the history of mathematics and proof by induction.

Radioactivity: Zone 8 –
This task uses a mathematical model to investigate half-life of radioactive substances. It involves number patterns, exponential functions, interpretation of graphs and probability.

Speed Graphs: Zone 8 -
This task provides students with an opportunity to create and interpret a mathematical model and to investigate linear functions and distance v time (speed) graphs.

Surface Area with Tricubes: Zone 8
This lesson provides a hands-on problem solving introduction to surface area, base area and volume.

ANNO'S MYSTERIOUS JAR

Specific teaching focus

To develop strategies for representing and solving a broader range of multiplicative situations, in this case, a problem involving the use of the Cartesian Product (or 'for each') idea to determine the number of outcomes or possibilities in a sample space.

Materials/resources required

- Anno, M. & Anno, M. (1983). *Anno's mysterious multiplying jar*. New York: Philomel Books.
- Coins, and dice (6-sided and 10-sided) as required
- Calculators

How to implement

1. Read the story to students. Discuss the magnitude of the progressive pattern as revealed in the story. The red dots at the conclusion of the story help to illustrate the number of countries, mountains, walled kingdoms,,, and so on. Discuss why the number of jars couldn't be represented by the red dots in the book.

2. Use a range of simpler problems (a strategy in itself, see below), to review different representations for problems of this type (Eg. tables, tree diagrams, symbolic expression involving multiplication). Examples of simpler problems: "how can you determine and illustrate the number of outcomes produced by tossing two coins, throwing 2 or more 6-sided dice, or throwing a 10-sided and a 6-sided dice, and so on."

3. Distribute A3 paper to small groups and invite them to represent the first few stages of the story and show how the number of possibilities might be expressed numerically. E.g.

4. Introduce factorial notation (ie, 1 x 2 x 3 x 4 = 4!), discuss the most efficient way of doing this a calculator, and invite groups to use factorial notation, calculators and/or pen and paper to evaluate for different items mentioned in the story. Then have students work in groups to consider what the number jars might be.

Follow up suggestions

Have students write their own story modelled on the Anno story (this does not need to follow the factorial pattern of the story). Students can present a tree diagram and formal notion together with their own creative text.

WORKING GROUPS

Specific teaching focus

To develop algebraic reasoning and representation strategies to solve problems involving multiplicative relationships.

Materials/resources required

- Working in Groups Problem (written on board, see below)
- Butcher's Paper

How to Implement

1. Distribute Butcher's paper and pose the following problem to students:

> **A maths teacher always likes to have her class working together in groups but no matter if she suggests her students work in groups of 2, 3 or 4 there is always one student on their own. How many students might there be in this teacher's class?**

2. Students may work in pairs or small groups to solve the problem.

3. Encourage students to approach the problem systematically and to clearly explain the strategies they use. Students should think of a range of possibilities. Ask students, "What is the most likely number of students in the class?" (E.g. Our group thought of all the multiples of 2, 3 and 4 and added 1 and wrote them down. Then we looked to see which numbers were in all the lists. Groups of two: 3, 5, 7, 9, 13, 15… Groups of three: 4, 7, 10, 13, 16, 19, 22… and Groups of four: 5, 9, 13, 17, 21…etc)

4. Discuss different strategies and how these might be represented.

Follow up suggestions

Have students write, share and solve their own 'Guess my Number' problems. Eg. "I am thinking of a number. When I multiply it by 3, subtract 13, halve the result and then divide by 2, I get 2. What's my number?" Discuss how these type of situations can be modelled using symbolic expressions and equations, and how they can be solved using a 'working backwards' strategy.

Further examples

"I bet you can't guess my number. It's a decimal fraction and it's 3 times as much as my dog is old. My dog is half my age and I am 12 years and 4 months old."

"Jim asked Sarah to tell him her age. She replied, "If you divide my age by 3, you will get the same answer as when you divide 75 by my age." How old is Sarah?"

"Find six consecutive numbers which add to 81."

PROBLEM SOLVING

Specific teaching focus

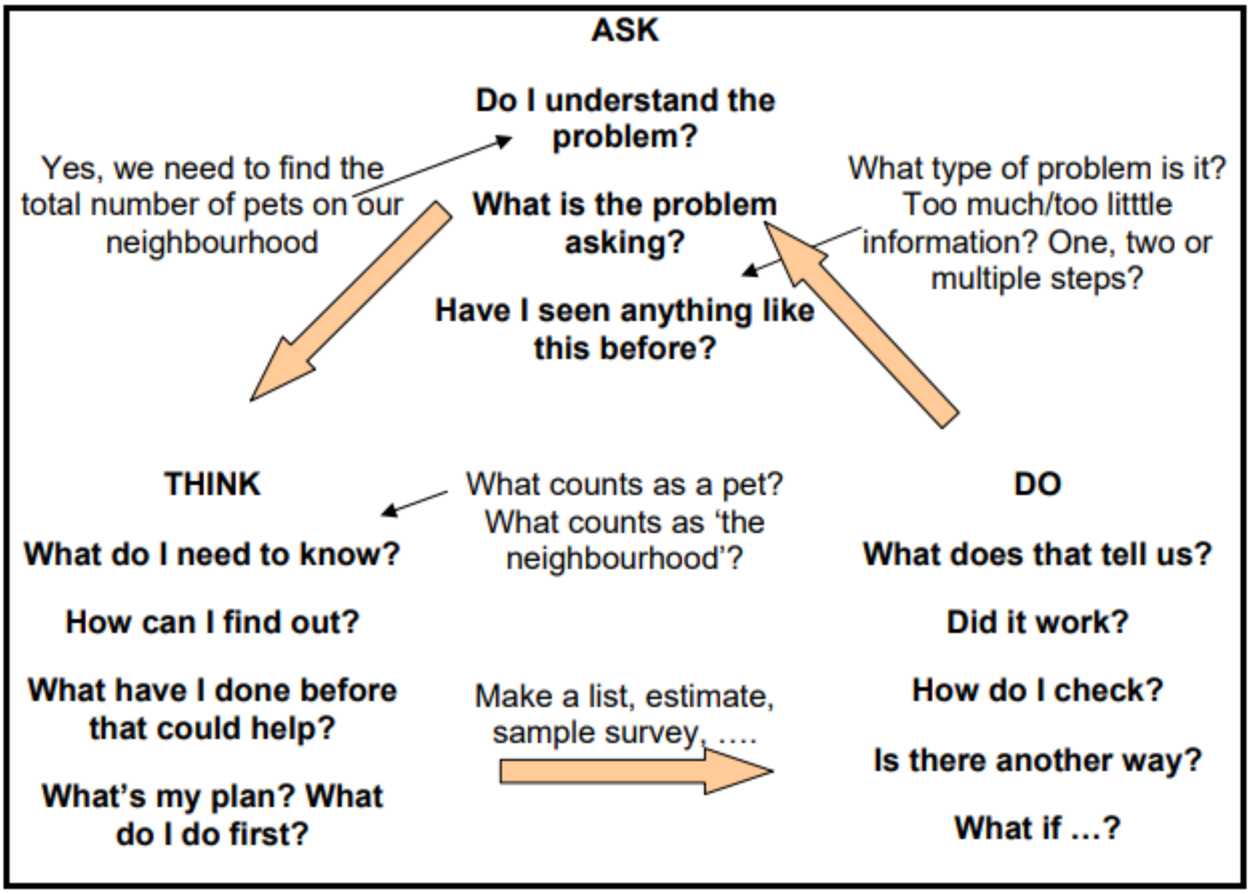
To develop an awareness of problem solving as a process which can be facilitated by recognising problem type and appropriate strategies.

Materials/resources required

- An Ask-Think-Do poster
- Copy The TYREing Trip from Melbourne to Perth worksheet (see below)

How to implement

1. Amend as required and use the Ask-Think-Do poster to brainstorm ideas and strategies in relation to the problem, “How many pets are there in the neighbourhood?” E.g.



2. Distribute worksheet and repeat this process for the “The TYREing Trip from Melbourne to Perth” problem. Emphasise the need to be sure that the problem is understood, that some thinking about problem type and what is already known is useful, and make a list of possible strategies.

3. Students work in pairs or small groups keeping track of the questions they ask and the strategies they use in their own “Ask-think-Do” poster. For the tyre problem students will need to realise that both empirical and metric units are used in the problem and a conversion is needed. Supply this to students when requested.

The TYREing Trip from Melbourne to Perth

A tyre manufacturer was testing two new tyre designs by driving two test cars identical in all respects except tyre design from Melbourne to Perth.

The diameters of the tyres were identical when fitted (both had a diameter of $25\frac{3}{8}$ **inches** at the start of the trial).

The cars were to travel from Melbourne to Perth by exactly the same route and to maintain the same speed at all times during the trip. The aim of the test was to see which tyre design resulted in less tyre wear over the course of the trial.

During the trial a communication error caused one driver (car B) to take the wrong route to Perth. This car drove a total of 4135 km on its journey which was $\frac{1}{5}$ longer than the distance traveled by Car A which took the designated route.

At the end of the trip the tyre diameters of both cars were measured again with the following results: car A diameter = 645.9**mm**; car B = 642.5**mm**.

Tyre manufacturers express tyre performance as percentage decrease in diameter per 1000km road contact (which means how many kilometres any one bit of the tyre was in contact with the road).

Calculate tyre performance for each car to determine which tyre design resulted in better tyre performance.

4. Guide students if necessary on
   - what is meant by distance travelled by car A
   - what is meant by percentage difference in diameters of tyres for both cars
   - how to relate the circumference of a tyre to the distance travelled
   - how to best organise the information calculated throughout the problem solution to keep track of it

5. Groups share solutions by explaining and justifying the process and strategies they used.

RECOGNISING PROBLEM TYPE

Specific teaching focus

To develop an awareness of problem solving as a process which can be facilitated by recognising problem type and appropriate strategies.

Materials/resources required

- Produce multiple copies of the Recognise Problem Type worksheet (see below)

How to implement

1. Discuss how problems vary, making a list on the board or on an overhead transparency (eg, too much, too little information, different number of steps involved, solution strategy very clear or unclear) and possible responses (E.g. or too much information — eliminate options, identify what is needed; for too little information - estimate, measure, make an assumption, research to find what is needed; for multiple steps – develop a systematic plan, keep records; and where solution strategies are not evident – make a table, explore possibilities, draw a diagram, use if … then reasoning). Talk about routine problems as well, that is, where just enough information is given and the solution strategy is fairly clear. E.g. “James bought 3.5 kg of bananas for $9.75/kg, how much did he spend on bananas?”

2. Distribute worksheet, students work in small groups to classify problems according to types described above:

Question	Classification
1	One step, solution strategy clear (if answered in terms of cost /kg), not enough information if interpreted as the core of a single apple, need to estimate, research to find number of apples/kg
2	Too much information, may involve one or two steps
3	Too little information (do not know mass of 1 bag), becomes multiple step once this is known/estimated or this could be determined for multiple bag sizes using a table or functional relationship
4	Too much information, multiple-steps
5	Too little information, solution strategy unclear, need to identify what is required and accommodate by estimating, assuming, using a table etc
6	Becomes routine once Q.5 resolved, multiple steps
7	Not enough information, solution strategy unclear, requires use of strategies such as make a table or eliminate options
8	Routine, multiple step problem

3. Discuss which problems can be grouped together and the benefits of doing this, e.g, if they are similar in type they are likely to require similar strategies and methods to solve them.

4. Once categorisation is agreed upon, students solve one from each category and share solution strategies.

Recognise the problem type and use the ASK-THINK-DO procedure.

1. **The mass of an apple core is about 8% of the mass of an apple. If apples are $1.75 a kilogram, how much are you paying for the core?**

2. **A potato grower estimates that the amount of dirt on the potatoes he takes to market accounts for 3% of their total mass. If he sold a 1.44 tonne load of potatoes for $331, how much would the potatoes be worth without the dirt?**

3. **If a greengrocer bought 3 bags of carrots for $36, how much should she sell them for, per kilogram, to make a 15% profit?**

4. **To sell the remaining 13.5 kg of mushrooms before they were too old, a greengrocer decided to put them on special at $4.29 per kilogram. If they had been selling previously for $5.50 per kilogram, what percentage reduction was this?**

5. **Ann and Jim's parents were giving a dinner party. For the main course they decided to serve roast beef, baked potatoes, honey-glazed carrots, buttered pumpkin and fresh beans. Ann and Jim were asked to buy the vegetables. How much of each vegetable should they buy?**

6. **How much change would they get from $50 if they took advantage of these specials?**

TODAY'S SPECIALS	
Apples	**2kg for $2.45**
Bananas	**$1.97 / kg**
Beans	**$2.67 / kg**
Broccoli	**$1.95 / kg**
Carrots	**89c / kg**
Lettuce	**89c each**
Mushrooms	**$4.95 / kg**
Peppers	**$3.79 / kg**
Potatoes	**5kg for $2.85**
Pumpkin	**$3.65 / kg**
Strawberries 2 punnets for $1.80	

7. **Kristy had to buy 2 kg of potatoes. How much change did she receive if she paid with a single note?**

8. **Greg bought ½ kg mushrooms, 2 kg carrots, 1 ½ kg beans, a lettuce and some strawberries. If he paid $12.76, how many punnets of strawberries did he buy?**

Follow up suggestion:

Rate problems are a good example of the advantages of recognising problem type and the importance of representation. There are many problems like the tank problem below which can be explored and solved on the basis of what is the same and what is different. Eg. “What did we do for the tank problem? Is that relevant here? How? Remember when we … for the tank problem” etc. Record the tank problem (see below) on the board or on a worksheet:

> ***Using my tap, it takes 6 minutes to fill our water tank. Using the neighbour’s hose it takes 9 minutes. How long would it take if I used both the tap and the hose?***

Discuss what information is provided and what is needed. Estimate the time it will take and the reasons why. Explore possible representations and the need to use or decide on a comparable base, e.g, how full after 1 minute if tap used? How full after 1 minute if hose used? Some students may want to work with 3 mins as the unit given the numbers involved which leads to a more elegant solution in this case, e.g,

After 3 mins, the tap has half-filled the tank and the hose has filled it an additional third, ie, After 3 mins the tank is 5/6 full, dividing 3 minutes by 5 gives the time required to fill the tank a further sixth, ie, 36 seconds, so the combined efforts of the hose and the tap will take 3 mins and 36 secs to fill the tank.

A similar problem that could be explored following this one, is “The excavator at the mine uses a litre of fuel every 3 mins. The truck uses a litre of fuel every 6 mins. How long will 100 litres of fuel last if both machines are being used?”

Another set of problems that are good for this purpose are those based on the well-known handshake problem, eg, 5 people at a party all shake hands with each other. How many handshakes will there be? This classical problem can be varied by changing the numbers and the context, eg.,

> ***At a New Year’s Eve party, each person in the room kissed every other person in the room once. By the end of the night there had been 190 kisses, how many people were in the room?***

Matchstick problems also provide a generic set of problems that can be solved more easily once the issue of representation has been recognised.

Problems involving two variables and two unknowns like the classical farmyard problem also lend themselves to generic solution strategies ranging from ‘guess and check’, through ‘if … then’ reasoning, to simultaneous equations.

Two examples are given below, both can be solved by simultaneous equations but the first is actually more elegantly solved by ‘if … then’ reasoning (Eg. assume all are spiders)

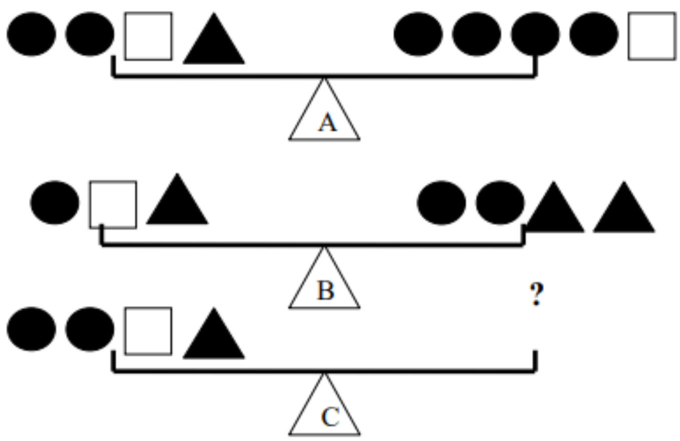
A mad scientist has a collection of spiders and beetles. His electronic floor sensor tells him that there are 174 legs altogether. The infra-red machine tells him that there are 26 bodies. How many beetles and how many spiders are there?

Three ducks and two ducklings weigh 32 kg. Four ducks and three ducklings weigh 44 kg. All ducks weigh the same and all ducklings weigh the same. What is the weight of two ducks and one duckling?

Another set of problems that lend themselves to easier solutions once the type of problem and appropriate forms of representations are recognised are balance problems.

These take a variety of forms but two are offered below.

"How many spheres must be placed on the right side of Scale C to make it balance?"



"An archaeologist found some ancient numbers written as follows:"