GROWING Mathematically

Multiplicative Thinking

Teaching Tasks (Zone 7)

Growing Mathematically is a project funded by the Australian Federal Government. It builds on materials that were developed as part of the RMIT Reframing Mathematical Futures II and the Scaffolding Numeracy in the Middle Years (SNMY) projects. SNMY content is currently hosted by the Victorian Department of Education.

Zone 1 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 7.

reSolve

Cartoon Counting: (more generalised understanding of place value)
Modelling Motion: (Strategies for recognising proportion problems)
Bar Model Method Sequence: (Strategies for recognising proportion problems)

Maths300

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Ice Cream Flavours: Zones 4 – 8
This task investigates the number of ice-creams that can be made given any number of flavours, any number of scoops and whether or not repeats are allowed.

Cracked Tiles: Zones 5 – 8
This task is one of several where geometric patterns lead to algebraic investigations and generalisations. This particular task uses rectangular arrays of tiles when investigating how many tiles need to be replaced when an electrician lays a cable along the diagonal of the rectangle.

Division Boxes: Zones 5 – 8
This task uses divisibility tests to develop a strategy for solving a divisibility problem. It quickly becomes an open-ended investigation. There is software to support the investigation.

Eric the Sheep: Zones 5 – 8
This task involves students in identifying a surprising pattern that can be linked to the physical structure of the problem. The pattern can be described in words or algebraically and can be extended to include step functions, domain and range.

Factorgrams: Zones 5 – 7
This task uses an interesting visual way to illustrate the factor relationships that exist within numbers. As well as the practise of division and multiplication facts, there is considerable opportunity for exploring prime factors. In addition several problem solving situations are presented.

Factors: Zones 5 – 8
There is a rule which can tell you how many factors exist for any number. This investigation is designed to both uncover that rule and see the logic behind it. The investigation helps develop a holistic understanding of how factors and prime factors are interconnected.

Newspaper Pathways: Zones 5 – 7
This task combines the estimation and calculation of length using large numbers, measurement and conversion of units, reading and applying map scales, with possible extensions into mass and surface area.

Painted Rods: Zones 5 – 7
Students use Cuisenaire rods or blocks to model simple cases, collect data from these and generalise the results into a rule. The first challenge is to work out the painted area for a rod 100 units long? A central aspect of the lesson is explaining and justifying the rule.

Snail Trail: Zones 5 – 7
This task sees a snail determined to climb out of a well sets out at a steady speed, but needs to rest after a given time. During the rest period it slips back a given amount. The challenge is to decide when it will escape. The puzzle can help students develop logical skills, however a world of algebra opens up by exploring the effect of changing the many variables involved.

The Mushroom Hunt: Zones 5 – 8
This task can be tackled at every level from an exploration of doubling, to an introduction to powers and indices, to the concept of binary numbers, to an investigation of multiplicative (exponential-like) growth. At each level application of problem solving strategies is required.

Unseen Triangles: Zones 5 – 7
In this task as a visual pattern develops students are encouraged to predict based on the pattern and then devise and explain a rule to extend it.

Garden Beds: Zones 6 – 8
This task has a very rich context from which many mathematical concepts can be explored. The mathematics of counting, area and perimeter, pattern and algebra ideas are all very evident. Seeing the construction of the garden in different ways leads into explaining patterns using algebra rules, equivalence of algebraic expressions, expanding brackets and collecting terms.

Heads and Legs: Zones 6 – 8
In this task students are asked to find how many of each type of animal there are given the number of heads and the number of legs they have collectively. The task can be solved in a variety of ways from drawing animals, or using the accompanying software to solving simultaneous equations.

Match Triangles: Zones 6 – 7
This task takes a simple problem which opens the door to visual and symbolic algebra, substitution, solving equations, linear and graphical algebra and more.

Walk the Plank: Zones 6 – 7
This outdoor game employs physical involvement to help students construct an image of what such operations on integers could mean. This task is mostly about integer arithmetic, however, especially because of the software, there is a second fruitful path which can be followed to investigate statistics and probability.

Protons and Anti-Protons: Zones 6 – 7
This task builds a valid, useable and mathematically sound model of positive and negative numbers. The emphasis of the lesson is on the personal construction of the concept. Time spent on this makes the development of the integer arithmetic skills easier.

Algebra Charts: Zones 7 – 8
This task builds on the Number Charts task and involves algebraic simplification and using the distributive property to factorise and expand algebraic expressions. It seamlessly links the inverse operations.

Baby in the Car: Zones 7 – 8
This task uses mathematical modelling to investigate the surface area to volume ratio.

Country Maps: Zones 7 – 8
This task investigates area and ratios in the context of comparing the areas of the states of Australia to each other.

Staircases: Zones 6 – 8
This task uses the visual pattern of the steps in a staircase. The discovery of the pattern opens the door to further algebra and to a visual representation of the pattern in graphical form, as well as the generalisation of the triangular numbers.

Twelve Days of Christmas: Zones 7 – 8

This task investigates the number of presents from the popular Twelve Days of Christmas song. It includes triangular numbers, square numbers and extends into quadratic functions.

MULTIPLE PATTERNS

Learning Goal

To recognise and apply multiplication and division to solve multi-step problems and recognise patterns in a series of numbers.

1. Look at the array of numbers on the Multiple Patterns 1 and use a marker to circle one number in each row and column. You will circle four numbers in total (no more than one in each row and column.)

2. Multiply the four numbers you have circled in your book without the use of a calculator.

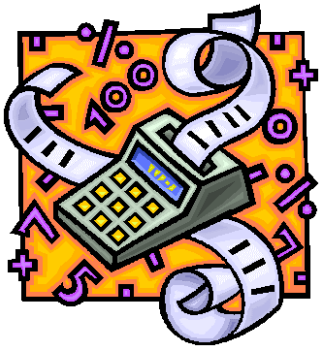
3. Wipe your circles off the laminate and select another four numbers using the same rules. Repeat step 3 with these numbers.

4. Compare your answers and record an observation in your book of what you notice about the two number sets.

5. Compare your answers with someone else's answers and record an observation in your book of what you notice about the four number sets.

6. Study the array of numbers. What patterns do you notice? Record this information in your book.

7. Is there any number that stands out in the array? If so, record which number and why it stands out using evidence to support your answer.



Extension

Follow the same steps (1-8) with one or both of the sets on the Multiple Patterns 2.

Multiple Patterns 1

12	21	3	15
32	42	8	40
16	28	4	20
36	63	9	45

Multiple Patterns 2

0.45	1.00	6.00	5.40
1.75	0.20	1.60	18.0
7.20	12.0	0.50	0.30
2.70	3.20	0.10	1.50

-12	35	12	-3
9	10	-28	2
-6	20	-8	4
-21	-15	-16	7

WORKING OUT VALUE FOR MONEY

Specific teaching focus

To develop strategies to recognise and apply multiplication and division in a broader range of situations including ratio and proportion (in particular the use of proportion in determining value for money.

Materials/resources required

- Supermarket shopping catalogues

How to implement

1. Provide students with a selection of shopping catalogues from various food outlets.

2. Students work in small groups to determine the best value for money for a selection of items in the catalogues.

3. Select an item that appears in a number of catalogues in a variety of sizes and prices. (E.g. at Outlet One 500g of Nodoz Coffee is $9.95, at Outlet Two 250g of Nodoz Coffee is $6.25 and at Outlet Three 750g of Nodoz Coffee is $12.50).

4. Pose the question, “Which of these is the best value for money?”

5. Groups discuss how they might solve the problem and record, justify and explain their solution.

6. Groups find other examples of ‘value for money’ in their catalogues and present their selection and solution to the other groups.

MAX'S MATCHSTICKS

Goal

To recognise and formally describe patterns involving all four operations.

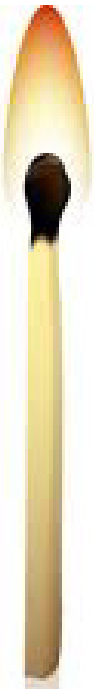
Steps

1. As a group look at the diagram on Max's Matchsticks 1 and decide if Max, Di, Sergio and Leanne have arrived at the same solution. Write the general rule that each person established into your book.

2. In pairs, choose one person's strategy and write in words in your book how the strategy is applied.

3. Choosing one strategy determine how many matchsticks would be needed to make:

    1. a. 5 cells b. 12 cells c. 27 cells d. 110 cells



4. Show your working out in your book and check each answer using another person's strategy.

5. As a group, investigate when the number of cells changes, which numbers change and which numbers stay the same for each strategy.

6. Explain in your own words (and copy into your book) why certain numbers change and others stay the same to make the rule work.

7. Max's strategy can be explained in terms of the number of cells as:

    $M = 2(n+1) + (n-1)$ where M = number of matches and n = number of cells

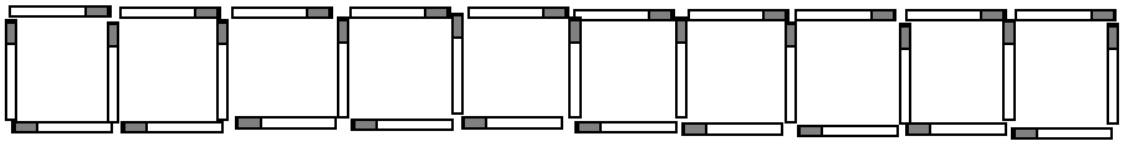
    Prove this works by substituting the number of cells and matches into the equation.

8. As a group try and determine the general rule for each of the other strategies in terms of M and n.

Extension

Describe the patterns on Max's Matchsticks 2 and describe the patterns you see in as many ways as possible. Write a general rule for each of the patterns in terms of M and n.

Max’s Matchsticks 1



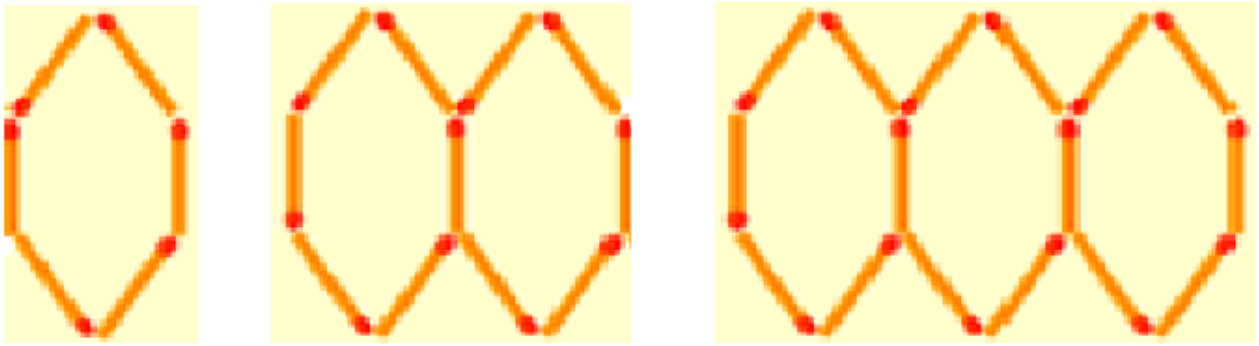
Max’s solution:
11 x 2 + 9

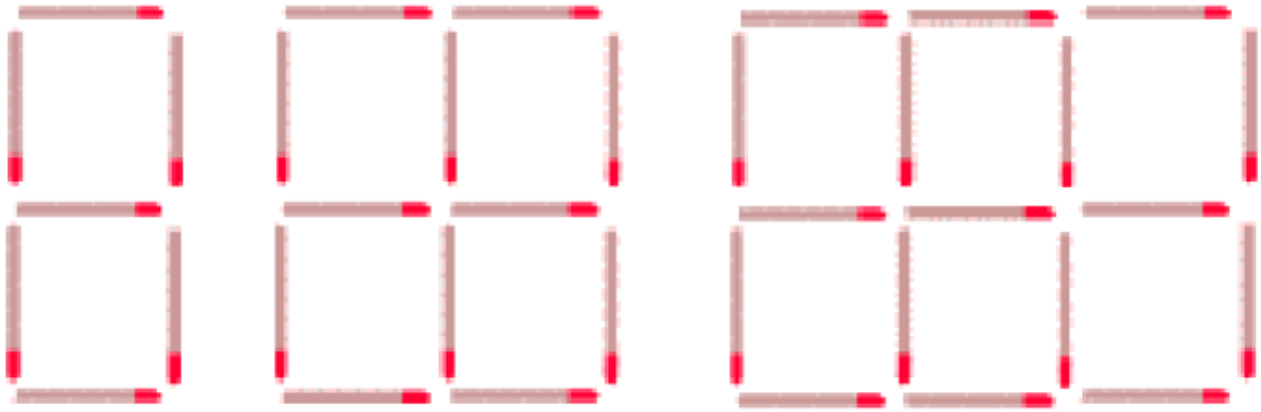
Di’s solution:
10 x 3 + 1

Sergio’s
solution:
4 x 10 - 9

Leanne’s
solution:
4 + (9 x 3)

Max’s Matchsticks 2





COMBINING SPEEDS

Specific teaching focus

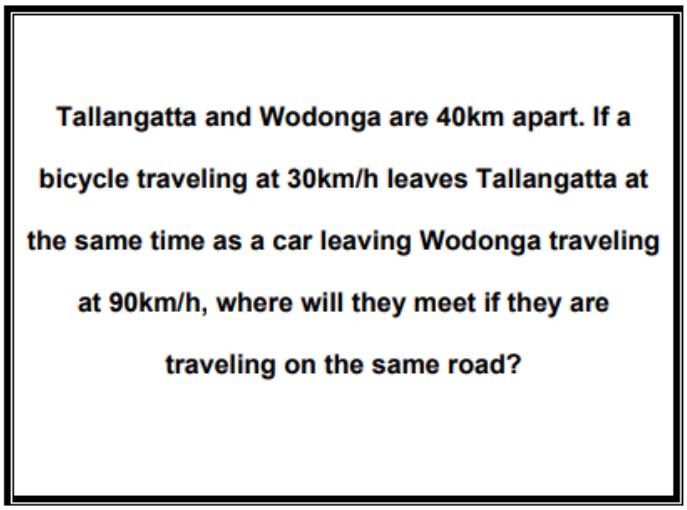
To develop strategies to recognise and apply multiplication and division in a broader range of situations and strategies for dealing with more complex problems involving derived measures in this case rate (speed).

Materials/resources required

- Tallangatta to Wodonga Problem (written on board, see below)

How to implement

1. Pose the following problem to students:



2. Students may work in pairs or small groups to solve the problem. Encourage students to draw a picture showing the two towns and their distance apart (E.g. on a number line) to facilitate their thinking. Students should record, justify and explain their solutions.

3. It may also be necessary for students to think about the speed the vehicles are traveling that is, the car is traveling 3 times as fast as the bicycle, (therefore it will travel 3 times further in the same amount of time). Encourage student to show this, E.g. as a ratio, 3:1 for every 3 distances the car travels the bicycle will travel 1 distance. (This gives 4 distances in total – so divide the distance between the two towns, 40km, into 4 equal parts of 10km each. The bicycle would travel 10km and the car 30km in the same time, so the vehicles would meet 10km from Tallangatta).

Follow up suggestions

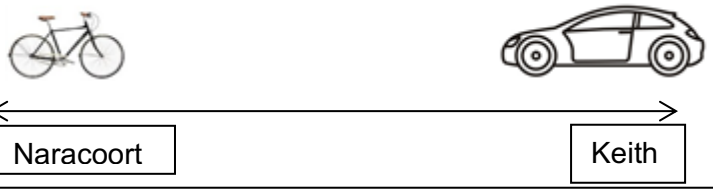
Pose another scenario: “Melbourne and Sydney are about 900km apart. If a motorcycle traveling at 120km/h leaves Sydney at the same time as a bus leaving Melbourne traveling at 90km/h, where will they meet?”

COMBINING SPEEDS 2

Learning goal

Solving word problems by using ratios.

Naracoorte and Keith are 100 km apart. If a bicycle travelling at 30 km/h leaves Naracoorte at the same time as a car travelling at 90km/h leaves Keith, where will they meet if they are travelling on the same road?



1. Draw your own picture showing the two towns and their distance apart (e.g. on a correctly scaled number line).

2. Solve the problem and record your thinking below.

3. Compare your answers with another group/student. Did they use the same strategy? Record any different strategies found.

4. Using a similar strategy, solve the following problem and record your thinking below.

Naracoorte and Melbourne are about 550 km apart. If a motorcycle travelling at 110km/h leaves Melbourne at the same time as a bus leaving Naracoorte travelling at 90km/h,

HOW MANY SQUARES?

Learning goal

Solving problems by recognising and dealing with patterns.

1. Work in a group, of no more than three. Collect 25 square tiles/blocks.

2. Use four of the tiles to make a 2 by 2 square. How many squares are formed altogether?

3. Now make a 3 by 3 square and count the total number of squares formed altogether.
   Don't forget to count the 2 by 2, and 1 by 1 squares.

4. Can you predict the total number of squares in a 4 by 4 square?

5. Make the 4 by 4 square and check whether your predication was correct or not.

6. Using your prediction for the 4 by 4 square, predict the number of squares in a 5 by 5 square.

7. Compare your answers with another group. Did they use the same strategy?
   Record any different strategies found.

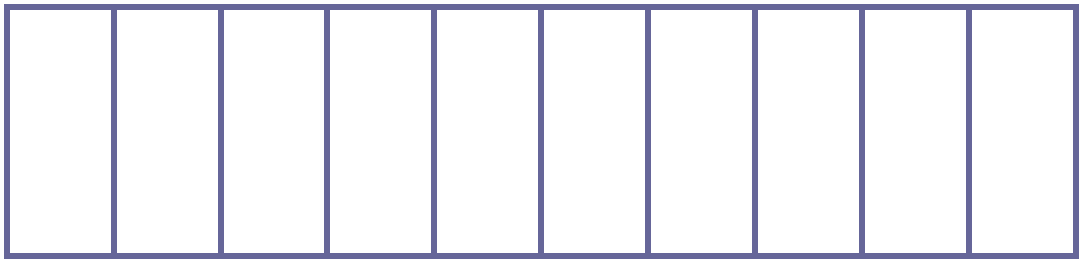
8. Imagine if you had not constructed the earlier squares, how could you represent the strategy as an algebraic equation? Consider using *S to represent any square with sides S by S*, and *N to represent the number of squares formed*. Find N.

MATCHSTICKS MAYHEM

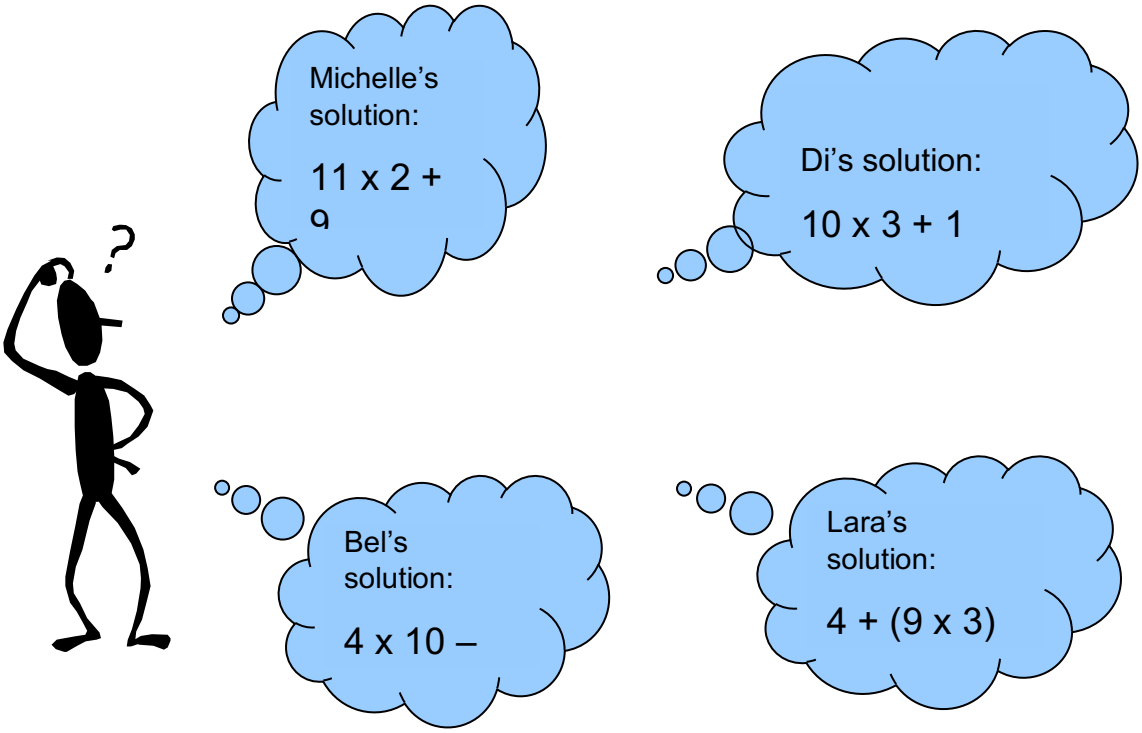
Learning goal

Solving problems by recognising and describing patterns.

1. Without counting each matchstick, work out how many matchsticks were needed to make the 10 squares shown below.



2. Have a look at the strategies used by the 4 different people below. Comment on whether each of these strategies are correct.



3. How many matchsticks would be needed to make 5 squares, 12 squares, 27 squares?

4. Explain how you think each of the 4 people arrived at her strategy?

USING RATIOS

Learning goal

Solving word problems by using ratios.

It takes Josie 60 minutes to walk the circuit called the “Naracoorte Creek Walk”.

This is approximately 4 km in length.

1. Josie is planning to walk to the Naracoorte Caves, 15 km from her home in Naracoorte. Use ratios to calculate how long it will take Josie, if she walks at the same pace (speed)?

2. If Josie leaves at 8:30 am, what time will she arrive at the Naracoorte caves?

3. If she has lunch at the caves and does a cave tour, taking 2 hours altogether, what time will it be when she gets home?

4. Josie also enjoys riding a bike. She can ride to Kybybolite, 20 km from Naracoorte, in 75 minutes. How long would it take her to ride to Keith, 105 km from Naracoorte?

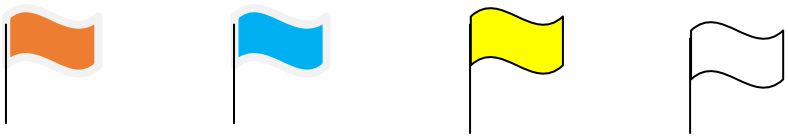
5. Josie is interested in riding her bike to Adelaide. If she rides from 8am till 5pm, with a total of two hours for breaks each day, how long will it take her? Adelaide is 335km from Naracoorte.

FLAGS FROM A SHIP

Learning goal

Solving problems efficiently when dealing with possible combinations.

You are on a ship and have four flags: one red, one blue, one yellow and one white.



Messages can be displayed from the mast by hoisting the flags in different arrangements. Each message you make must use the four flags.

1. Record as many messages as you can with the flags you have so that each message has the flags in a different order. Record your flag messages on the other side of this sheet.

2. What is the largest number of different messages that can be made with four different flags?

3. Discuss your results with another group.

4. Can you find a quicker way to solve this problem without drawing each of the combination of flags.

5. Is there a way to write this as an algebraic expression? Hint: make *F* the total number of flags.

PIZZAS

Learning goal

Solving word problems by using ratios.

As fundraiser on Sports Day the SGC is planning to make and sell mini pizzas. They will be making a pizza called “Energy plus”, which will have meat, vegies and cheese in the ratio of 2 : 3 : 1, by weight.

1. The SGC has asked you to calculate the total number of grams of meat, vegies and cheese required to make 12kg of pizzas. (*Ignore the weight of the pizza bases.*) Clearly show your calculations below.

2. After some advertising, the SGC realise that the “Energy Plus” mini pizzas are going to be very popular. They would like you to factor in the pizza bases also.
   The ratio of base : meat : vegies : cheese = 3 : 4 : 6 : 2

   They ask you to calculate the quantities required for making 20kg of pizzas.

3. Compare your answers with another group/student. Did they use the same strategy? Record any different strategies found.

Which soft drink is the best value for money?

Coca-Cola Soft Drink 30x375mL

Was $34.45, Save $16.45
$18.00 each

Pepsi, Solo or Schweppes Soft Drink 24x375mL

Was $21.00, Save $8.50
$12.50 each

V Energy Drink 4x275mL

Was $9.35, Save $3.35
$6.00 each

Coca-Cola, Fanta or Sprite Soft Drink 10x375mL

Was $12.30, Save $5.30
$7.00 each

Which shampoo/conditioner is the best value for money?

Pantene 3 Minute Miracle Conditioner 180mL

Was $6.99, Save $3.50
$3.49 each

Pantene Shampoo or Conditioner 350mL

Was $6.99, Save $3.50
$3.49 each

Garnier Fructis Shampoo or Conditioner 700mL

Was $12.00, Save $6.00
$6.00 each

Organic Care Shampoo or Conditioner 400mL

Was $3.50, Save $1.05
$2.45 each

Pantene Shampoo or Conditioner 900mL

Was $15.70, Save $7.85
$7.85 each

Which cheese is the best value for money?

Mainland Cheddar Vintage 125g

$4.00

Bega Tasty Cheese 500g

$8.60

Mersey Valley Original Vintage Cheese 180g

$6.00

Mainland Epicure Cheese 200g

$6.00

Which coffee is the best value for money?

Gloria Jeans Ground Coffee Smooth Classic Blend 200g

$8.80

Chicco D'oro Delta Ground Coffee Chicco Doro 1kg

$12.00

SPECIAL

Vittoria Coffee Oro Coffee Beans Oro 500g

$12.00

Was $20.00

Jarrah Coffee Vienna Cinnamon Latte 250g

$6.00

ORANGE JUICE TASK

Every year the seventh grade students at Langston Hughes School go to an outdoor education camp. During the week-long trip, the students study nature and participate in recreational activities. Everyone pitches in to help with the cooking and cleanup.

Ari and Maria are in charge of making orange juice for all the campers. They make the juice by mixing water and orange juice concentrate. to find the mix that tastes best, Ari and Maria decided to test some recipes on a few of their friends.

Ari and Maria tested four juice mixes.

Mix A 2 cups concentrate 3 cups cold water	**Mix B** 1 cup concentrate 4 cups cold water
Mix C 4 cups concentrate 8 cups cold water	**Mix D** 3 cups concentrate 5 cups cold water

1. Which recipe will make the juice that is most "orangey"? Explain your answer

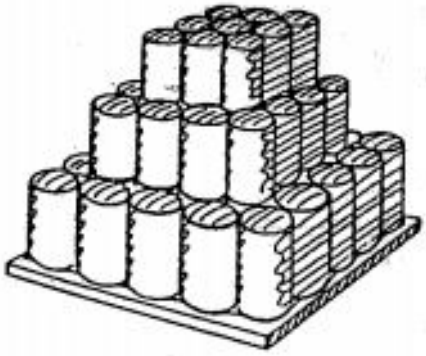
2. Which recipe will make the juice that is the least "orangey"? Explain your answer.

3. Assume that each camper will get half a cup of juice. For each recipe, how much concentrate and how much water are needed to make juice for 240 campers? Explain your answer.

Lappin et al, Connected Mathematics Project, 1998

The Supermarket Packer

An experienced supermarket packer knew that the available shelf space was 5 cans deep and 3 cans high. She also knew that for the same 'footprint' of 25 cans (5 rows of 5 cans) she could stack 50 cans using the 'pyramid' method, 60 cans using the 'steps' method, and 75 cans using the 'maximum' stock method.

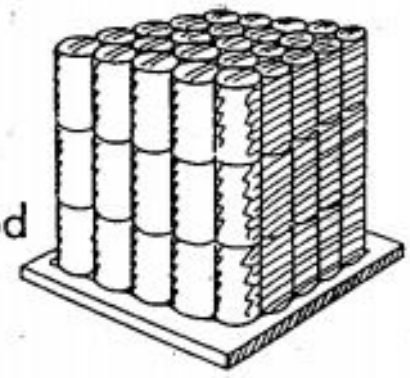


the 'pyramid' method

Investigate what these differences would be if the 'footprint' was 5 rows of 9 cans.

Is there a general 'rule of thumb' that might be used to guide the packer's stacking?

the 'steps' method



the 'maximum stock' method