GROWING Mathematically

Multiplicative Thinking

Teaching Tasks (Zone 6)

Growing Mathematically is a project funded by the Australian Federal Government. It builds on materials that were developed as part of the RMIT Reframing Mathematical Futures II and the Scaffolding Numeracy in the Middle Years (SNMY) projects. SNMY content is currently hosted by the Victorian Department of Education.

Zone 6 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 6.

reSolve

Reaction Time: (Hundredths as a new place value part)

Chicken Boxes: (Notion of a variable)

Assessing Reasoning: Painted Cube (Notion of a variable)

Directed Number: (Integers)

Maths300

Tackling Times Tables: Zones 2 – 6
This task builds the conceptual background using an array model and explores the number bonds associated with multiplication facts and the distributive law. It can be extended into algebraic reasoning using the structure of the array model.

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Multiplication in a Table Format: Zones 3 – 6
The technique presented in this task provides a non-standard algorithm for multiplication that has a strong theoretical connection to arrays and the distributive property. It progresses naturally into expanding and factorising algebraic expressions.

Fraction Estimation: Zones 3 – 6
In this task students are challenged to improve their estimates of fractions by considering what constitutes the whole, how many equal parts it is to be divided into and how many of the parts need to be chosen. After students have done this with physical models there is software using various representations of fractions to use.

4-Arm Shapes: Zones 4 – 6
This visual activity links a counting task with multiplicative patterns which may be described algebraically.

Bob's Buttons: Zones 4 – 6
This task involves working backwards from data about the number of equal groups and remainders to find the original number.

Chocolate Cake: Zones 4 – 6
This task uses an area-based model of fractions to investigate what happens when different numbers of chocolate cakes are cut into various fractions and how those fractions compare to each other.

First Principles Percent: Zones 4 – 6
This task uses simple ratios to provide a first principles' approach to understanding percentages.

Fractions to Decimals on a Rope: Zones 4 – 6
The focus of this task is to establish a strong conceptual understanding of equivalence when converting fractions to decimals. This understanding is built on concrete, visual experiences before students move to a formal algorithm.

Ice Cream Flavours: Zones 4 – 8
This task investigates the number of ice-creams that can be made given any number of flavours, any number of scoops and whether or not repeats are allowed.

Planets: Zones 4 – 6
This task uses large numbers, estimation and simple proportion to place the planets of the solar system along a rope.

This Goes with This: Zones 4 – 6
Rarely is there as powerful an illustration which makes important mathematical concepts and their integration so clear and understandable. The lesson uses the students' own survey data to demonstrate links between fractions and percentages, and strip graphs, circle graphs and pie charts.

Cracked Tiles: Zones 5 – 8
This task is one of several where geometric patterns lead to algebraic investigations and generalisations. This particular task uses rectangular arrays of tiles when investigating how many tiles need to be replaced when an electrician lays a cable along the diagonal of the rectangle.

Division Boxes: Zones 5 – 8
This task uses divisibility tests to develop a strategy for solving a divisibility problem. It quickly becomes an open-ended investigation. There is software to support the investigation.

Eric the Sheep: Zones 5 – 8
This task involves students in identifying a surprising pattern that can be linked to the physical structure of the problem. The pattern can be described in words or algebraically and can be extended to include step functions, domain and range.

Factorgrams: Zones 5 – 7
This task uses an interesting visual way to illustrate the factor relationships that exist within numbers. As well as the practise of division and multiplication facts, there is considerable opportunity for exploring prime factors. In addition several problem solving situations are presented.

Factors: Zones 5 – 8
There is a rule which can tell you how many factors exist for any number. This investigation is designed to both uncover that rule and see the logic behind it. The investigation helps develop a holistic understanding of how factors and prime factors are interconnected.

Fraction Magic Square: Zones 5 – 6
The unfamiliar fraction magic square is linked to the familiar 3 x 3 whole number example. Students refresh their fraction equivalence and addition and move on to use these skills to create their own magic square.

Goldbach’s Conjecture: Zones 5 – 6
The task is essentially about recognising prime numbers and it includes much skill practice in addition. However, the investigation and genuine history behind the challenge gives it a richness beyond just skill practice.

How Many People Can Stand? Zones 5 – 6
In this task the question is asked in the context of a story shell where the classroom becomes the 'standing room only' section of a venue, for the purposes of encouraging estimation and calculation of number and area using a variety of approaches, including ratio, to solving the problem.

Multo: Zones 4 – 6
This task involves designing a grid that will win a game of Multo. It involves knowledge of multiplication facts, factors and sample spaces.

Newspaper Pathways: Zones 5 – 7
This task combines the estimation and calculation of length using large numbers, measurement and conversion of units, reading and applying map scales, with possible extensions into mass and surface area.

Painted Rods: Zones 5 – 7
Students use Cuisenaire rods or blocks to model simple cases, collect data from these and generalise the results into a rule. The first challenge is to work out the painted area for a rod 100 units long? A central aspect of the lesson is explaining and justifying the rule.

Snail Trail: Zones 5 – 7
This task sees a snail determined to climb out of a well sets out at a steady speed, but needs to rest after a given time. During the rest period it slips back a given amount. The challenge is to decide when it will escape. The puzzle can help students develop logical skills, however a world of algebra opens up by exploring the effect of changing the many variables involved.

The Mushroom Hunt: Zones 5 – 8
This task can be tackled at every level from an exploration of doubling, to an introduction to powers and indices, to the concept of binary numbers, to an investigation of multiplicative (exponential-like) growth. At each level application of problem solving strategies is required.

Unseen Triangles: Zones 5 – 7
In this task as a visual pattern develops students are encouraged to predict based on the pattern and then devise and explain a rule to extend it.

Area of a Circle: Zones 5 – 6
The approach used in this task is a first principles iterative 'guess and check' method with a calculator, rather than the traditional algebraic method. The main background skill needed is knowing how to calculate the area of a *square*.

Area of a Triangle: Zones 5 – 6
This task is firmly aimed at the concept or first principles level, the challenge being to convert an irregular shape, namely a triangle, into the more familiar and regular rectangle. In doing this, emphasis is given to visualisation skills, concrete materials and the graphic animation capacity of the supporting software.

Garden Beds: Zones 6 – 8
This task has a very rich context from which many mathematical concepts can be explored. The mathematics of counting, area and perimeter, pattern and algebra ideas are all very evident. Seeing the construction of the garden in different ways leads into explaining patterns using algebra rules, equivalence of algebraic expressions, expanding brackets and collecting terms.

Heads and Legs: Zones 6 – 8
In this task students are asked to find how many of each type of animal there are given the number of heads and the number of legs they have collectively. The task can be solved in a variety of ways from drawing animals, or using the accompanying software to solving simultaneous equations.

Match Triangles: Zones 6 – 7
This task takes a simple problem which open the door to visual and symbolic algebra, substitution, solving equations, linear and graphical algebra and more.

Walk the Plank: Zones 6 – 7
This outdoor game employs physical involvement to help students construct an image of what such operations on integers could mean. This task is mostly about integer arithmetic, however, especially because of the software, there is a second fruitful path which can be followed to investigate statistics and probability.

Protons and Anti-Protons: Zones 6 – 7
This task builds a valid, useable and mathematically sound model of positive and negative numbers. The emphasis of the lesson is on the personal construction of the concept. Time spent on this makes the development of the integer arithmetic skills easier.

Staircases: Zones 6 – 8
This task uses the visual pattern of the steps in a staircase. The discovery of the pattern opens the door to further algebra and to a visual representation of the pattern in graphical form, as well as the generalisation of the triangular numbers.

BLOCK PATTERN FOR A QUILT

Specific teaching focus

To develop explanation and justification solution strategies for problems involving multiplication, division and proportion.

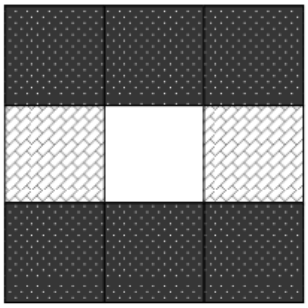
Materials/resources required

- Black, grey and white kinder squares

How to implement

1. Present the following problem:

Some children are making a quilt out of material in an art class. Each block is made up of 9 squares. To make this block you need 6 black squares, 2 grey squares and 1 white square. It looks like this.



This block uses 2 grey squares for every 6 black squares. How many black squares would you need if you had 6 grey squares?

2. Students use materials (Eg. kinder squares or similar) to illustrate and describe their thinking. E.g. “I have three times as many grey squares, so I need 3 times as many black squares.”

3. Showcase all the ways that students describe and justify their solutions.

MULTIPLYING WITH GRAPH PAPER

Specific teaching focus

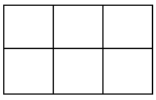
To introduce more efficient strategies and formal processes for multiplication of decimals based on sound place-value ideas.

Materials/resources required

- 1cm grid paper
- 1mm grid paper

How to implement

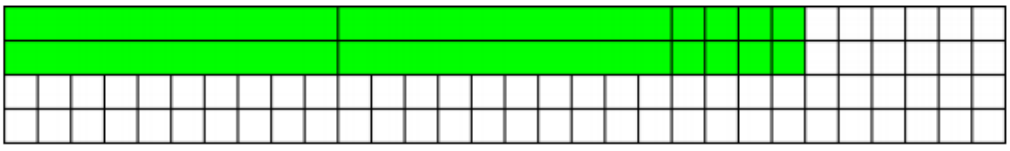
1. Provide each student with a sheet of 1cm grid paper and discuss with them how to show number facts using grid paper.



Eg. *"2 threes."*

Repeat with a few more whole number examples.

2. Then, using the 1mm grid paper, ask students to explore how they might show 2, two and 4 tenths (2 by 2.4) which should look like this:|



Give students time to explore, discuss and model their thinking for possible solutions.
E.g. 2 twos, 4; and 2 by 4 tenths, 8 tenths; so the answer is 4 and 8 tenths (4.8).

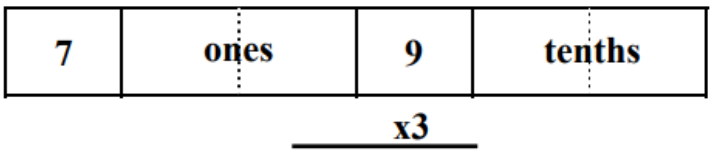
3. The following problems can be presented to students to document and solve in the same way.

3 by 3.6 3 by 2.8

4 by 2.2 4 by 5.3

Follow up suggestions

Number expanders can also be used to show decimal multiplication to support formal recording, e.g.:



Language to support formal recording:

- 3 ones by 9 tenths is 27 tenths, rename as 2 ones and 7 tenths, record 7 tenths with tenths and 2 ones to regroup.

- 3 ones by 7 ones is 21 ones and 2 more ones is 23 ones, record with the ones (solution 23.7)

SQUARE NUMBERS

Specific teaching focus

To develop the ability to recognise and describe more complex patterns through exploring square numbers.

Materials/resources required

- counters
- 1cm grid paper

How to implement

1. Pose the following investigation: “Justify why 9 and 36 are square numbers and 24 is not.”

2. Students work in small groups to share their understanding of what the investigation is about (E.g. what square numbers are).

3. Students use materials to illustrate and describe why 9 and 36 are square numbers and 24 is not.

4. Share justifications with other groups. In particular encourage students to identify the properties of square numbers. Eg. “I can arrange 9 counters and 36 counters as a square shape in the form of an array and I can’t do this with 24 counters.”

Follow up suggestions

Investigate the factors of odd and even square numbers. E.g. 25 (factors are 1, 25, 5) and 36 (factors are 1, 36, 6, 2, 18, 9, 4, 3, 12).

TARGET PRACTICE

You need: 4 ten-sided dice (different colours are useful but not essential) and a copy of the game sheet for each student.

To play:

1. Take turns to throw the dice.
2. Choose 3 of the numbers thrown to make a number as close to the target number as possible.
3. Mentally calculate 'how close' and record in the space provided.
4. The winner is the player with the smallest total.

Numbers thrown	TARGET	Number Made	How close?
	567		
	9.45		
	93.6		
	5.7		
	13.8		
	4		
	627		
	0.81		

TOTAL

TARGET PRACTICE

You need: 3 or 4 ten-sided dice (different colours are useful but not essential) and a copy of the game sheet for each student.

To play:

1. Take turns to throw the dice.
2. Choose 2 or 3 of the numbers thrown to make a number as close to the target number as possible.
3. Mentally calculate 'how close' and record in the space provided.
4. The winner is the player with the smallest total.

Numbers thrown	TARGET	Number Made	How close?

TOTAL