GROWING Mathematically

Multiplicative Thinking

Teaching Tasks
(Zone 5)

Growing Mathematically is a project funded by the Australian Federal Government. It builds on materials that were developed as part of the RMIT Reframing Mathematical Futures II and the Scaffolding Numeracy in the Middle Years (SNMY) projects. SNMY content is currently hosted by the Victorian Department of Education.

Zone 5 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 5.

reSolve

Multiplication: The Tiler (Area idea to support multi-digit multiplication)
Multiplication: reSolve Bakery (Area idea to support multi-digit multiplication)

Maths300

Cookie Count: Zones 2 – 5
This task introduces the idea of equal shares. Challenges occur at a range of levels with both whole numbers and fractions.

Tackling Times Tables: Zones 2 – 6
This task builds the conceptual background using an array model and explores the number bonds associated with multiplication facts and the distributive law. It can be extended into algebraic reasoning using the structure of the array model.

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Fractions and Fraction Charts: Zones 3 – 5
This task looks at fractions as discrete groups and then at fraction charts where the inverse operations are used to reason through a problem. Other tasks in the maths300 suite such as Fraction Estimation and Rectangle Fractions investigate visualising and other representations of fractions.

Multiplication in a Table Format: Zones 3 – 6
The technique presented in this task provides a non-standard algorithm for multiplication that has a strong theoretical connection to arrays and the distributive property. It progresses naturally into expanding and factorising algebraic expressions.

Fraction Estimation: Zones 3 – 6
In this task students are challenged to improve their estimates of fractions by considering what constitutes the whole, how many equal parts it is to be divided into and how many of the parts need to be chosen. After students have done this with physical models there is software using various representations of fractions to use.

4-Arm Shapes: Zones 4 – 6
This visual activity links a counting task with multiplicative patterns which may be described algebraically.

Bob's Buttons: Zones 4 – 6
This task involves working backwards from data about the number of equal groups and remainders to find the original number.

Chocolate Cake: Zones 4 – 6
This task uses an area-based model of fractions to investigate what happens when different numbers of chocolate cakes are cut into various fractions and how those fractions compare to each other.

First Principles Percent: Zones 4 – 6
This task uses simple ratios to provide a first principles' approach to understanding percentages.

Fractions to Decimals on a Rope: Zones 4 – 6
The focus of this task is to establish a strong conceptual understanding of equivalence when converting fractions to decimals. This understanding is built on concrete, visual experiences before students move to a formal algorithm.

Ice Cream Flavours: Zones 4 – 8
This task investigates the number of ice-creams that can be made given any number of flavours, any number of scoops and whether or not repeats are allowed.

Planets: Zones 4 – 6
This task uses large numbers, estimation and simple proportion to place the planets of the solar system along a rope.

Rectangle Fractions: Zones 4 – 5
In this task a rectangular array is the model which helps students visualise and conceptualise the arithmetic of fractions. A combination of everyday objects, concrete materials and software work together to help students understand fraction concepts and refine fraction skills.

This Goes with This: Zones 4 – 6
Rarely is there as powerful an illustration which makes important mathematical concepts and their integration so clear and understandable. The lesson uses the students' own survey data to demonstrate links between fractions and percentages, and strip graphs, circle graphs and pie charts.

Cracked Tiles: Zones 5 – 8
This task is one of several where geometric patterns lead to algebraic investigations and generalisations. This particular task uses rectangular arrays of tiles when investigating how many tiles need to be replaced when an electrician lays a cable along the diagonal of the rectangle.

Division Boxes: Zones 5 – 8
This task uses divisibility tests to develop a strategy for solving a divisibility problem. It quickly becomes an open-ended investigation. There is software to support the investigation.

Eric the Sheep: Zones 5 – 8
This task involves students in identifying a surprising pattern that can be linked to the physical structure of the problem. The pattern can be described in words or algebraically and can be extended to include step functions, domain and range.

Factorgrams: Zones 5 – 7
This task uses an interesting visual way to illustrate the factor relationships that exist within numbers. As well as the practise of division and multiplication facts, there is considerable opportunity for exploring prime factors. In addition several problem solving situations are presented.

Factors: Zones 5 – 8
There is a rule which can tell you how many factors exist for any number. This investigation is designed to both uncover that rule and see the logic behind it. The investigation helps develop a holistic understanding of how factors and prime factors are interconnected.

Fraction Magic Square: Zones 5 – 6
The unfamiliar fraction magic square is linked to the familiar 3 x 3 whole number example. Students refresh their fraction equivalence and addition and move on to use these skills to create their own magic square.

Goldbach's Conjecture: Zones 5 – 6
The task is essentially about recognising prime numbers and it includes much skill practice in addition. However, the investigation and genuine history behind the challenge gives it a richness beyond just skill practice.

How Many People Can Stand? Zones 5 – 6
In this task the question is asked in the context of a story shell where the classroom becomes the 'standing room only' section of a venue, for the purposes of encouraging estimation and calculation of number and area using a variety of approaches, including ratio, to solving the problem.

Licorice Factory: Zones 4 – 5
This task involves students identifying prime numbers by investigating factors and products.

Make a Million: Zones 3 – 5
This task investigates how big a million is.

Multo: Zones 4 – 6
This task involves designing a grid that will win a game of Multo. It involves knowledge of multiplication facts, factors and sample spaces.

Newspaper Pathways: Zones 5 – 7
This task combines the estimation and calculation of length using large numbers, measurement and conversion of units, reading and applying map scales, with possible extensions into mass and surface area.

Painted Rods: Zones 5 – 7
Students use Cuisenaire rods or blocks to model simple cases, collect data from these and generalise the results into a rule. The first challenge is to work out the painted area for a rod 100 units long? A central aspect of the lesson is explaining and justifying the rule.

Snail Trail: Zones 5 – 7
This task sees a snail determined to climb out of a well sets out at a steady speed, but needs to rest after a given time. During the rest period it slips back a given amount. The challenge is to decide when it will escape. The puzzle can help students develop logical skills, however a world of algebra opens up by exploring the effect of changing the many variables involved.

Tables for 25: Zones 4 – 5
This task asks students to find multiple solutions for arranging a class while satisfying certain conditions. Factors and primes become important as the problems are investigated.

The Mushroom Hunt: Zones 5 – 8
This task can be tackled at every level from an exploration of doubling, to an introduction to powers and indices, to the concept of binary numbers, to an investigation of multiplicative (exponential-like) growth. At each level application of problem solving strategies is required.

Unseen Triangles: Zones 5 – 7
In this task as a visual pattern develops students are encouraged to predict based on the pattern and then devise and explain a rule to extend it.

Area of a Circle: Zones 5 – 6
The approach used in this task is a first principles iterative 'guess and check' method with a calculator, rather than the traditional algebraic method. The main background skill needed is knowing how to calculate the area of a *square*.

Area of a Triangle: Zones 5 – 6
This task is firmly aimed at the concept or first principles level, the challenge being to convert an irregular shape, namely a triangle, into the more familiar and regular rectangle. In doing this, emphasis is given to visualisation skills, concrete materials and the graphic animation capacity of the supporting software.

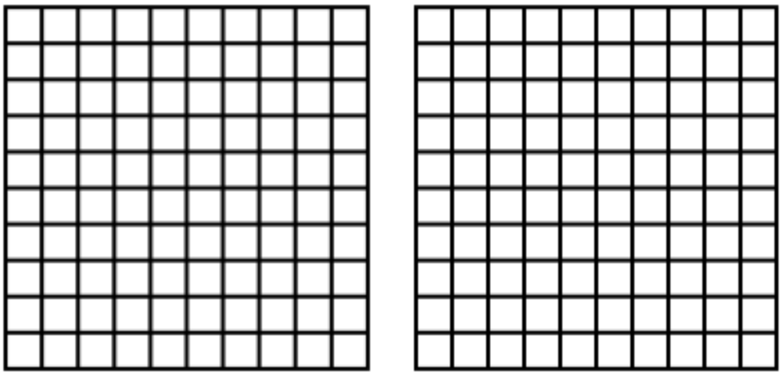
DASHING DECIMALS

Specific teaching focus

To introduce place-value ideas and strategies for tenths and hundredths; make, name, record.

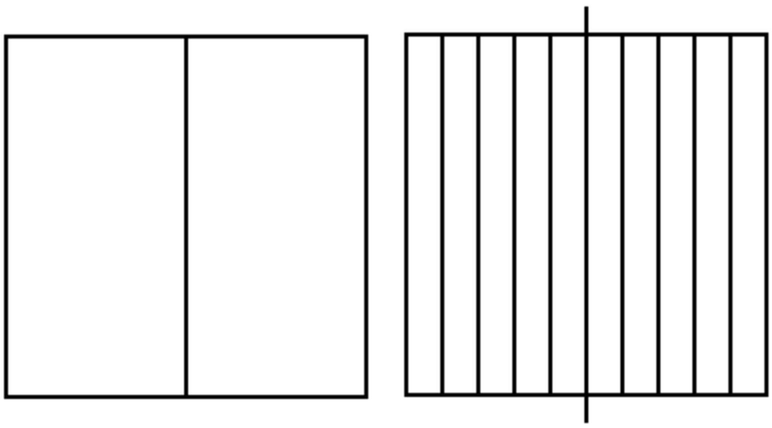
Materials/resources required

- Kinder squares
- Two different coloured 10 sided dice, for pairs of students
- Recording sheet with ten (10) 10 x 10 grids (see part example below)

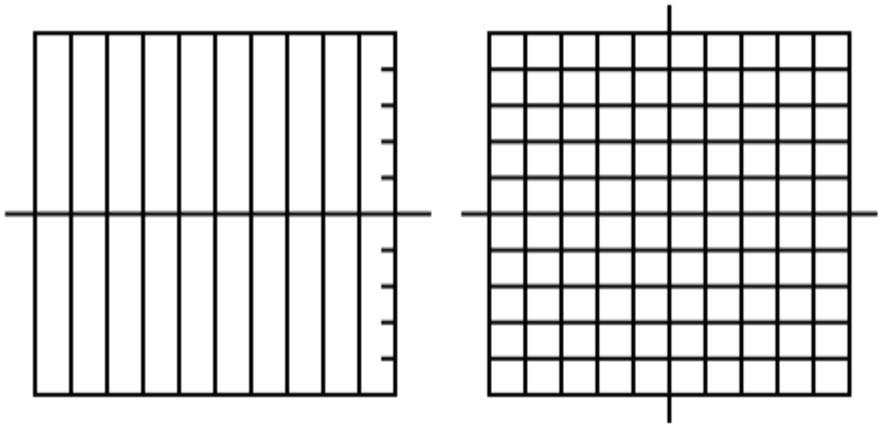


How to implement

1. Using kinder squares students partition their square to show tenths using halving then fifthing strategies (make sure students partition their square in one dimension only, see below)



2. Discuss with students how they could make hundredths from this. (E.g. tenths by tenths, or ten rows of ten, 100 parts gives hundredths)



Allow students time to create their own fraction diagram showing tenths and hundredths (noting each row or column is tenths while each square is hundredths).

3. Provide each student with a recording sheet containing ten (10) 10 x 10 grids. In pairs, students toss 2 different coloured 10 sided dice to play the game. Students determine which coloured dice will represent tenths and which will represent hundredths. Students take turns throwing the dice and shading in their grids accordingly (remind students that, tenths can be represented as columns or rows).

4. Students also record their throws as decimal fractions and fractions (i.e, if a 5 is thrown on the tenths dice and an 8 is thrown on the hundredths dice, student would record 0.58 and 58/100. If a 0 is thrown on the tenths dice and 7 is thrown on the hundredths dice, student would record 0.07 and 7/100). Game continues until one student has shaded all ten grids.

5. As students play the game, the teacher roves among the groups and asks, “How much have you shaded so far? How much do you have left to shade?”

DECIMAL COMPARISONS

Specific teaching focus

To introduce place-value ideas and strategies for ordering tenths and hundredths.

Materials/resources required

- 1 mm graph paper
- 2 metre measuring tapes
- Rope, pegs and blank cards

How to implement

1. Students work co-operatively in small groups (3-5 students) to find out how tall they are. Students document results and arrange these in order from shortest to tallest. Discuss the various ways in which this information can be recorded. E.g. 165 cm, 1 m 65 cm, 1.65 m.

2. Students then draw each of these measurements to scale using 1mm graph paper, where 1mm equals 1cm. Students can now compare their original ordering by comparing these with the scale drawings.

3. Model an open number line with rope and pegs, labelled 0 to 2. Record each student's height on a card and have students locate these on the rope. Students should justify their decisions by explaining their thinking. Eg. "I know that 165 cm is more than 3 quarters of the way along the rope."

4. Swap results with another group and record these heights on an open number line in their workbooks.

FACTOR FIND

Specific teaching focus

To introduce formal terminology associated with multiplication and division such as factor, product, divisor, multiplier.

Materials/resources required

- Counters

How to implement

1. Discuss with students how 8 counters can be represented as an array (2 rows of 4, 4 rows of 2, 8 rows of 1, 1 row of 8).

2. Discuss with students that the numbers 2 and 4, 1 and 8 are called factors of 8, that is quantities that divide a given number exactly.

3. Repeat the above, for collections of 12, 21, 36, 19, 72. Discuss the factors as students construct models in the form of arrays. Ask students how can we check that we have all the factors for a number? (this may lead to a systematic method as below).

   E.g. Possible systematic thinking to assist with solutions for the number 8.

   1 by what gives 8, 8, 1 eight

   2 by what gives 8, 4, 2 fours

   3 by what gives 8, something more than 2 but less than 3, therefore 3 isn't a factor.

   4 by what gives 8, 2. We have this pair already so these are all the factors, so there are 4 factors of 8, 1, 2, 4, and 8.

4. Students can use this method to determine the factors for a variety of numbers, going as high as is appropriate, based on students' understanding.

Follow up suggestions

This is only one way of working and introducing factors to students - teachers can also use factor trees and the game 'MULTO' (Maths300) to increase the understanding of factors.

CHOCOLATE PARTITIONING

Specific teaching focus

To introduce formal terminology associated with multiplication and division such as factor, product, divisor, multiplier.

Materials/resources required

- One 250g block of chocolate wrapped in newspaper
- 1cm grid paper

How to implement

1. Start with a block of chocolate (based on 250g block of Cadbury diary milk) wrapped in paper. Pose the question "How many pieces of chocolate might there be and how might these pieces be arranged." Have the children explore and record possible solutions. Eg. 5 rows of 6, 4 rows of 12 etc.

2. Once they have come up with a few possibilities, students select the solution they think is most accurate. Students draw this on grid paper.

3. Tell the children that there are 54 pieces (based on 250g block of Cadbury diary milk). What might the block look like? Explore, record and discuss. Reveal the block and check against student responses.

4. Now pose the following problem: "If a 250g block of chocolate has 54 pieces (9 sixes), what might a 400g look like?" Reasoning can be based on ratio and proportion and need not be totally accurate. (Eg. from calculations for 100g, a block would be approximately 22 pieces so 400g would be 88 pieces or a 250g block is 5/8 of a 400g block. A 400g block might be useful to check against).

DIVERSE DIMENSIONS

Specific teaching focus

To introduce formal terminology associated with multiplication and division such as factor, product, divisor, multiplier and introduce metacognitive strategies to support problem comprehension, strategy monitoring/checking, and interpretation of outcomes relevant to context.

Materials/resources required

- 1 cm grid paper if required for explanation

How to implement

1. Pose the following problem to students and then lead them through a discussion guided by the solution below, explaining any new terms that are introduced, reviewing concepts of area and perimeter. “A rectangle has a perimeter of 28 cm. What might the area be?”

2. Allow students to explore possible solutions using the 1cm grid paper.

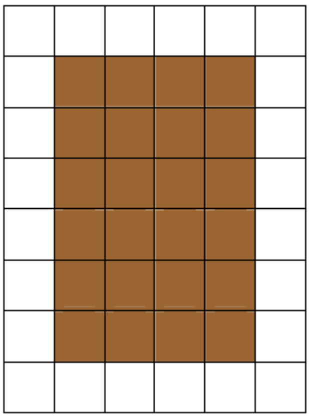
3. One possible solution could be: Think, half the perimeter is a length and a width together which would be 14cm (length x width is area, remind students of the concept of arrays). Think of all the possible combinations to make 14 (Eg. if the width is 1cm the length would be 13 cm, see table below).

Width cm	Length cm	Area cm^2
1	13	13
2	12	24
3	11	33
4	10	40
5	9	45
6	8	48
7	7	49

So, there are seven different dimension combinations that will give a perimeter of 28cm. 4.
Discuss, “Why does 7 by 7 give a greater area than 1 by 13?”

Follow up suggestions

Pose the following problem to students:
If the problem above referred to a block of chocolate with 28 pieces around the outside, how many pieces would be left if the outside pieces were removed? (See diagram.) Describe and discuss for each of the alternative dimensions referred to above. Encourage students to model their approach using grid paper or tiles.

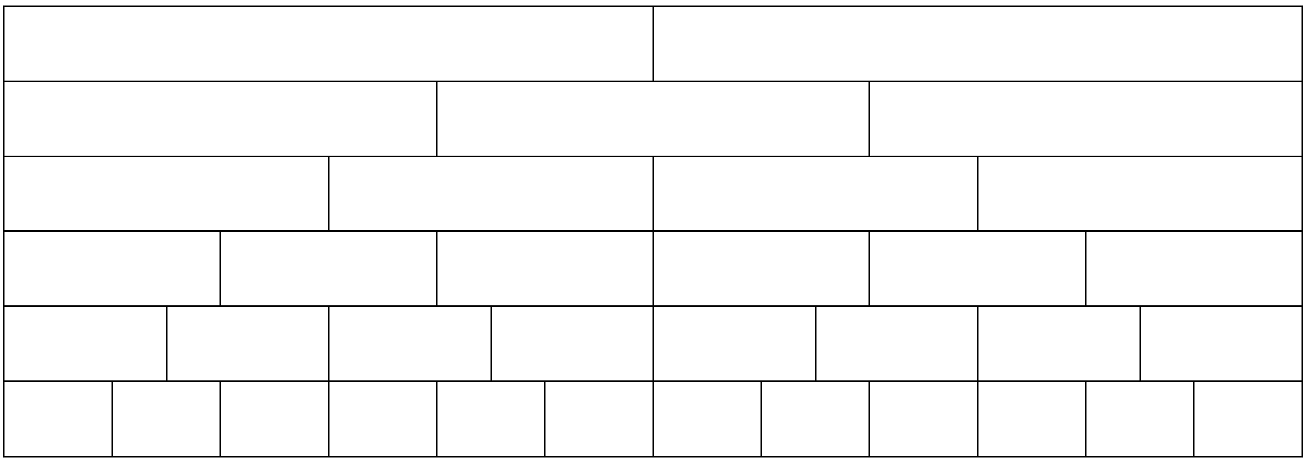


HOW MANY WHOLES CAN WE MAKE?

Learning goal

To introduce strategies for renaming fractions (equivalent fractions)

1. Form a pair and collect a card set labelled “How many wholes?”
2. Place the cards face down, in 2 piles, between you and your partner.
3. Take it in turns to turn over one card from each pile, to make a fraction.
4. Shade this fraction on your fraction wall below, where <u>each row represents one whole</u>.
5. Note that if your cards show “3 sixths”, you could shade “1 half”, or “6 twelfths” or any other equivalent fraction.
6. If you cannot shade the fraction given by the cards, you miss a turn.
    a. *The winner is the first to shade in all of their fraction wall!*



7. If you were giving someone advice on how to win this game, what strategies might you suggest?

Fraction cards for How Many Wholes?

thirds	thirds	thirds
thirds	thirds	thirds
quarters	quarters	quarters
quarters	quarters	quarters
halves	halves	halves
sixths	sixths	sixths

sixths	sixths	sixths
eighths	eighths	eighths
eighths	eighths	eighths
eighths	eighths	eighths
twelfths	twelfths	twelfths
twelfths	twelfths	twelfths

1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6

7	7	7
8	8	8
1	1	1
2	2	2
3	3	3
4	4	4

BRANCHING OUT WITH TREE DIAGRAMS

Specific teaching focus

To develop more efficient strategies for representing and solving an expanded range of Cartesian product problems involving three or more variables and tree diagram representations.

Materials/resources required

- Sandwich Task (written on board, see below)
- Gina's Pizza Task (written on board, see below)
- Develop a Blank Tree Diagram sheet (see below for example)

How to implement

1. Teacher poses the Sandwich Task problem. Students work individually or in pairs to solve the problem. Students share individual group ideas with a view to identifying the most efficient strategies, lists, tables etc. Then model process for tree diagram strategy.

Sandwich Task

Nick's dad is making him a sandwich to take to school for lunch. He can choose one ingredient from each category:

- **White or brown bread**
- **Margarine or butter**
- **Tomato, cheese or lettuce**

How many different sandwiches can he make?

2. Teacher poses Gina's Pizza problem. Students work independently using a tree diagram to solve it. When everyone is finished students share their answers and their thinking. Notice key features of tree diagrams that help with efficiency. E.g. 2 x 2 x 3. Discuss the "for each" idea to develop the multiplicative nature of finding the total number of different combinations. (E.g. for each type of bread, you can use two different spreads, 4 different combinations (2 x 2) and for each of these combinations you can choose three different fillings,12 different combinations in all (2 x 2 x 3).

Gina's Pizza

You've just begun working at Gina's Pizza Shop. She's a fussy boss and you know you will have to work quickly and accurately to keep your job.

Customers can order pizzas with:

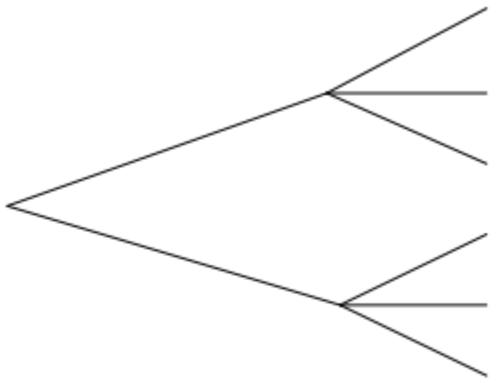
- **a thick or thin crust**
- **tomato or barbecue sauce**
- **salami or olive or mushroom toppings**

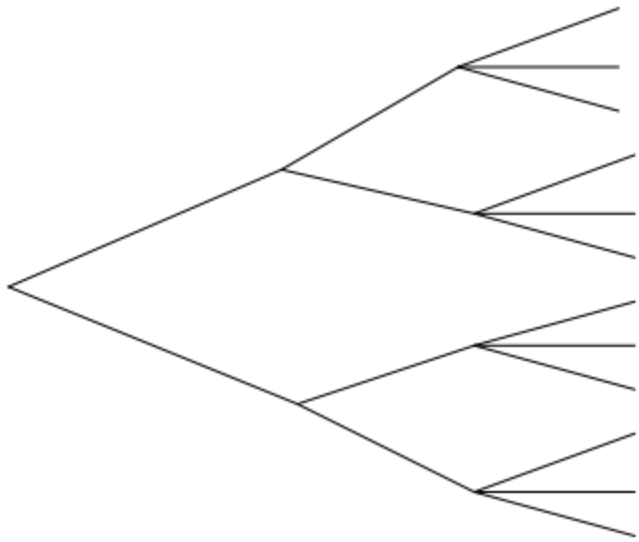
A family comes into the shop. How many different pizzas are possible?

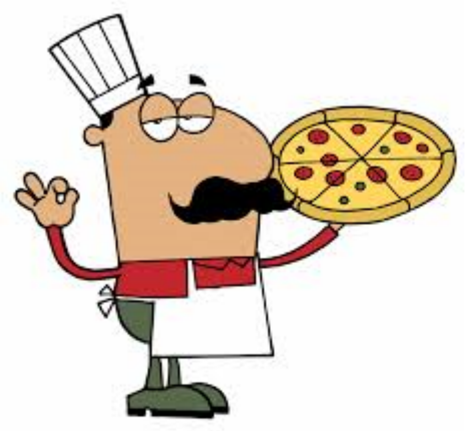
3. Give students a Blank Tree Diagram sheet as illustrated below, a choice of two can be provided according to student ability.

4. Ask students to write a story or scenario that relates to the tree diagram. Share students' stories and tree diagrams.

5. Discuss the 'for each' idea for multiplication and how it differs from the 'equal groups' idea.







PAPA LAMBINI'S PIZZA SHOP ORDERS

You've just begun working as a Pizza Chef at Papa Lambini's and you know that you'll need to work quickly and accurately to keep your job.

Customers can order pizzas with the following ingredients:

- Thin Crust, Deep Pan or Stuffed Crust
- Beef, Pepperoni or Chicken
- Barbecue Sauce or Tomato Sauce
- Pineapple or not
- Onion or not
- Capsicum or not
- Olives or not

1. A family comes in to the shop and orders a pizza. How many different pizzas can possibly be made?

2. Pizzas can be ordered in small, large or family size. How many different possible pizzas can be made now?

3. Family size pizzas are cooked in a square tray, other than thin crust which is cooked in a circular tray, as are all of the small and large pizzas.

A family comes in to the store and orders a pizza. Unfortunately, when you pick up the order docket you notice it is torn and some of the information is missing. On the piece of docket you can see that the order is for a chicken pizza with pineapple and onion but no capsicum. You have no idea the rest of the order and the lady who took the order has gone home. You heard the children say they are glad they are getting a square pizza because you get more pieces. How many different pizzas would you need to cook to ensure that the family gets the pizza they ordered?

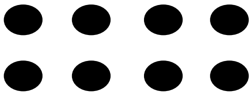
FACTOR FIND

Learning Goal

To introduce **formal terminology** with multiplication and division such as factor and product.

Collect some counters or blocks.

1. Take 8 counters. How can these be represented to show an array (e.g. 2 rows of 4).



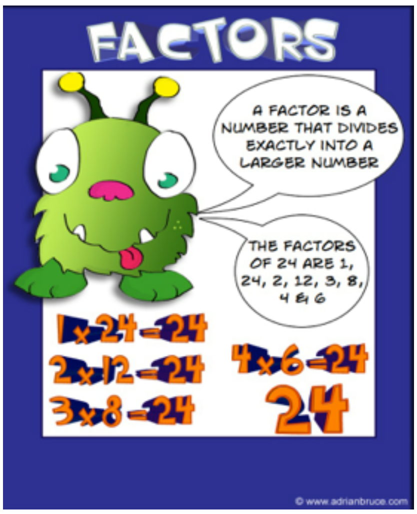
2. The numbers in this array (2 and 4) are called *Factors*, which are quantities (numbers) that can divide a given number exactly. What are the other factors in 8?

3. Repeat for collections of 12, 21, 36, 19, 72. Record your results in the table.

Number	Factors
8	
12	
21	
36	
19	
72	

4. How can we be sure that we have every factor for each number? Discuss your answers with a partner and record in your book.

5. Choose two more numbers to add to the table, and find their factors.



:

JACK'S MODEL CARS

Aim

To practice multiplication, division, ratio and Cartesian product concepts.

Jack is making simple model cars. The car is made up of 1 body, 4 wheels and 2 headlights.

Show calculations for each question in your workbook.

1. If Jack wanted to make 5 model cards, how many bodies, wheels and headlights would he need?

2. Jack gets out all the materials he can find from his craft box. He wants to use up as much of what he has. Jack finds he has 16 wheels, 4 bodies and 8 headlights.

    a. How many <u>complete</u> model cars can he make?

    b. Will he have any material left over?

    c. If so, what is left over?

    d. Why can't he use this left over material?

3. Using the strategies you have used for multiplication this year, work out how many bodies, headlights and wheels Jack need for 58 model cars.

4. How many complete model cars could you Jack make with 29 wheels, 8 bodies and 13 headlights?

   Show all your working and explain your answer in as much detail as possible.

5. Jack wants to paint his model cars. One day he worked out he could paint 2 cars with 7ml of paint. Work out how much paint he would need for 10 cars.

6. Jack found another 70ml of paint in the back of the cupboard. How many model cars could he paint?

7. Jack decides he would like to make a variety of different looking model cars. At the craft shop, there are different choices for the body and wheels.

Body	**Wheels**	**Headlights**
Square	Small	Round
Round	Medium	
	Large	

How many different model cars are possible to make? Show your working.

8. The next time Jack visited the craft shop, he found they had introduced a square headlight. How many different model cars can Jack make now?

9. Jack wanted to make his model cars even better. He decided to add a spoiler on to the car. The next time Jack visited the craft shop, he saw that they had two versions of a spoiler. How many different model cars can Jack make now?

10. Jack then decided he might paint the cars different colours. The craft shop had model car paint in 3 choices: yellow, red or green. How many different model cars can Jack make now?

FACTOR CROSS

Find common factors and place them in the table so that:

- The numbers in each row multiply to give the number alongside the row
- The numbers in each column multiply to give the number above the column

	12	16	9
24			
18			
4			

HINT: Write down all the common factors including 1

	14	10	21
4			
49			
15			

	40	15	42
7			
9			
6			
25			

Try making one up on your own

MULTIPLE PATTERNS

12	21	3	15
32	42	8	40
16	28	4	20
36	63	9	45

Circle one number from each column and one number in each row

You cannot choose more than one number from any row or any column.

Use any method you like but it will need to be accurate

Multiply the numbers together.

Record the product. Repeat with another set of numbers or compare your answer with someone else's answer.

What do you notice?

Study the table of numbers. What patterns do you notice?

Is there an error? If so, where and why?