GROWING Mathematically

Multiplicative Thinking

Teaching Tasks (Zone 4)

Growing Mathematically is a project funded by the Australian Federal Government. It builds on materials that were developed as part of the RMIT Reframing Mathematical Futures II and the Scaffolding Numeracy in the Middle Years (SNMY) projects. SNMY content is currently hosted by the Victorian Department of Education.

Zone 4 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 4.

reSolve

Multiplication: Cartesian Product (More efficient strategies for multiplication and division of larger whole numbers)

Maths300

Cookie Count: Zones 2 – 5
This task introduces the idea of equal shares. Challenges occur at a range of levels with both whole numbers and fractions.

Tackling Times Tables: Zones 2 – 6
This task builds the conceptual background using an array model and explores the number bonds associated with multiplication facts and the distributive law. It can be extended into algebraic reasoning using the structure of the array model.

Number Charts: Zones 1– 4
This task covers additive strategies in Zones 1 and 2 and multiplicative strategies in Zones 3 and 4. The task seamlessly connects the inverse operations and uses reasoning strategies. Connects to Fractions and Fraction Charts and Algebra Charts.

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Fractions and Fraction Charts: Zones 3 – 5
This task looks at fractions as discrete groups and then at fraction charts where the inverse operations are used to reason through a problem. Other tasks in the maths300 suite such as Fraction Estimation and Rectangle Fractions investigate visualising and other representations of fractions.

Multiplication in a Table Format: Zones 3 – 6
The technique presented in this task provides a non-standard algorithm for multiplication that has a strong theoretical connection to arrays and the distributive property. It progresses naturally into expanding and factorising algebraic expressions.

Fraction Estimation: Zones 3 – 6
In this task students are challenged to improve their estimates of fractions by considering what constitutes the whole, how many equal parts it is to be divided into and how many of the parts need to be chosen. After students have done this with physical models there is software using various representations of fractions to use.

Rod Mats: Zones 3 – 4
This task explores equivalent fractions and operations involving fractions. The whole can be varied to represent different challenges.

Times 99: Zones 3 – 4
This task explores mental computation strategies for multiplying by 99 and then extends to include multiplying by numbers close to 99.

4-Arm Shapes: Zones 4 – 6
This visual activity links a counting task with multiplicative patterns which may be described algebraically.

Bob's Buttons: Zones 4 – 6
This task involves working backwards from data about the number of equal groups and remainders to find the original number.

Chocolate Cake: Zones 4 – 6
This task uses an area-based model of fractions to investigate what happens when different numbers of chocolate cakes are cut into various fractions and how those fractions compare to each other.

First Principles Percent: Zones 4 – 6
This task uses simple ratios to provide a first principles' approach to understanding percentages.

Fractions to Decimals on a Rope: Zones 4 – 6
The focus of this task is to establish a strong conceptual understanding of equivalence when converting fractions to decimals. This understanding is built on concrete, visual experiences before students move to a formal algorithm.

Ice Cream Flavours: Zones 4 – 8
This task investigates the number of ice-creams that can be made given any number of flavours, any number of scoops and whether or not repeats are allowed.

Planets: Zones 4 – 6
This task uses large numbers, estimation and simple proportion to place the planets of the solar system along a rope.

Rectangle Fractions: Zones 4 – 5
In this task a rectangular array is the model which helps students visualise and conceptualise the arithmetic of fractions. A combination of everyday objects, concrete materials and software work together to help students understand fraction concepts and refine fraction skills.

This Goes with This: Zones 4 – 6
Rarely is there as powerful an illustration which makes important mathematical concepts and their integration so clear and understandable. The lesson uses the students' own survey data to demonstrate links between fractions and percentages, and strip graphs, circle graphs and pie charts.

Licorice Factory: Zones 4 – 5
This task involves students identifying prime numbers by investigating factors and products.

Make a Million: Zones 3 – 5
This task investigates how big a million is.

Multo: Zones 4 – 6
This task involves designing a grid that will win a game of Multo. It involves knowledge of multiplication facts, factors and sample spaces.

Tables for 25: Zones 4 – 5
This task asks students to find multiple solutions for arranging a class while satisfying certain conditions. Factors and primes become important as the problems are investigated.

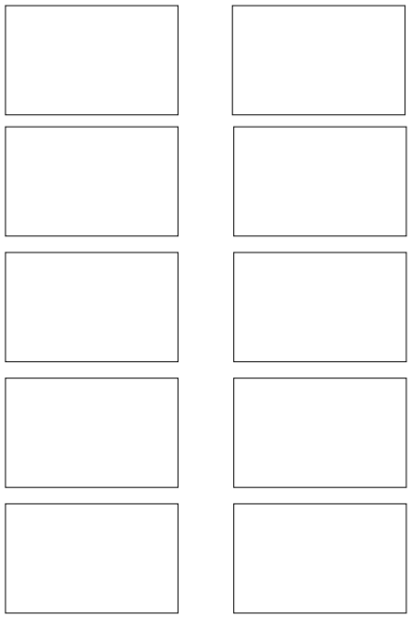
IDENTIFYING FRACTIONS

Specific teaching focus

To introduce partitioning continuous quantities by developing the halving, thirding, fifthing strategies, and establishing how to use this knowledge to construct fraction diagrams and representations for common fractions and decimals.

Materials/resources required

• Worksheet with approximately 10 rectangle templates (see below)



How to implement

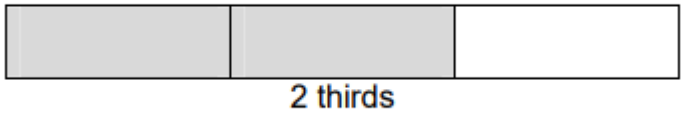
1. Ask students to suggest some names for fractions and record these on the board in words. E.g. 2 thirds.

2. Discuss the thirding strategy with students. E.g. think 1 third is smaller than 1 half. Draw a rectangle on the board and ask if anyone can partition to show 2 thirds.

3. Invite a student to demonstrate partitioning the rectangle.

4. Repeat for fifths using the ‘fifthing’ strategy.

5. Using the worksheet mentioned above, the teacher suggests a number of different fractions for the students to partition on their rectangles and write the fraction in words underneath (see below).



2 thirds

Encourage students to use the oral language associated with naming fractions and to articulate strategies used when partitioning.

SHARING PIZZAS

Specific teaching focus

To introduce partitioning continuous quantities through using the halving, thirding, fifthing strategies, and how to use this knowledge to construct fraction diagrams and representations for common fractions and decimals.

Materials/resources required

- Paper circles
- Scissors
- Play dough
- Icy pole sticks
- Rolling pins

How to implement

1. Have students work in groups of four. With paper circles representing pizzas ask students to show how they would share one pizza with their group (play dough, icy pole sticks, rolling pins can also be used to model these situations). Encourage students to describe their thinking to create equal shares and to name the share e.g, each person gets 1 quarter of the pizza.

2. Ask students to show how they would share two pizzas with their group.

3. Repeat for three, four and five pizzas.

FRACTION ACTION

Specific teaching focus

To introduce partitioning continuous quantities through using the halving, thirding, fifthing strategies, and how to use this knowledge to construct fraction diagrams and representations for common fractions and decimals.

Materials/resources required

- Lengths of rope about 2-3 metres
- Sets of 6-8 different coloured plastic pegs
- Cards labelled with various simple common fractions as appropriate (e.g. $\frac{3}{4}, 1\frac{1}{2}$ etc.)

How to implement

The ideas underpinning this targeted intervention are the number of parts (how many), equal parts (how much) to create the whole.

1. Stretch out a length of rope (held between two students) and ask a couple of students to place a peg where they think half is (the halfway point of the rope).

2. Other students should indicate which peg they think is closest to representing half and suggest ways of checking eg, students suggest that the tape needs to be folded into half in order to check accuracy.

Follow up suggestions

- Once students have experienced thirding and fifthing via paper folding, this activity can be repeated for thirding and fifthing strategies.
- Students place cards along a rope labelled 0 to 2. Eg. $\frac{2}{3}, 1\frac{1}{2}, \frac{3}{4}, \frac{5}{5}$, $1\frac{2}{5}$.

CONTINUOUS QUANTITIES

Specific teaching focus

To introduce partitioning continuous quantities through using the halving, thirding, fifthing strategies.

Materials/resources required

- Rope
- Coloured clothes pegs

How to implement

1. The rope can represent a range of whole numbers at various intervals on a number line, e.g., 0 to 2, or 3 to 5, or 7 to 9 etc.

2. Ask students to place a different coloured peg at given intervals as appropriate, eg for the rope showing 3 to 5, first ask, what number would be placed at the half way point? Discuss result (four).

3. Ask students if the next peg is to be placed at $3\frac{1}{2}$, what strategies would you use? Discuss result (halving).

4. Repeat the above for $3\frac{1}{4}$, $4\frac{2}{3}$, $4\frac{2}{5}$ dealing with one fraction at a time. (Thirding and fifthing).

5. As students engage with the activity, focus discussion on: What are you thinking about, when deciding where to place the peg? How do you know that your spot is about right? How might you check?

EXPLORING HALVING STRATEGIES

Specific teaching focus

To introduce partitioning continuous quantities through using the halving strategies.

Materials/resources required

- Paper streamers

How to implement

1. Teacher provides each student with a streamer and says that the streamer is equal to 48.

2. Ask students to show half of their streamer and what that point represents, (ie: half of 48, 24).

3. What happens when we halve the streamer again? Discuss four parts, quarters and how this can be described, eg, quarter of 48 and 3 quarters of 48. Share strategies and thinking for determining the value of these points, eg, think: 1 quarter of 48 is half of half of 48, so 12.

EXPLORING THIRDING STRATEGIES

Specific teaching focus

To introduce partitioning continuous quantities through using thirding strategies to develop concept of fraction as operator idea.

Materials/resources required

- Paper streamers
- Whiteboard
- Clothes peg and string

How to implement

1. Teacher provides each student with a streamer and says that the streamer is equal to 48.

2. Ask students to partition their streamer into three parts (thirds) and work with a partner or in small groups to discuss strategies for naming these points on the streamer. Share ideas with the larger working group. Eg. If I divide 48 by 3 that will tell me what 1 third of 48 is.

3. What happens when we halve the thirds? Have students work in a similar fashion as described above to discuss six parts, sixths and the numbers that would be represented at these points on the streamer. Eg. 8, 16, 24, 32, 40.

NAMING FRACTIONS

Specific teaching focus

To introduce partitioning continuous quantities through using the halving, thirding, fifthing strategies, and developing the knowledge of how to construct fraction diagrams and representations for common fractions and decimals.

Materials/resources required

- Paper streamers
- Writing instruments

How to implement

1. Distribute paper streamers to students, ask them to write 0 at the left and 100% at the right of their streamer.

2. Without measuring formally, ask students to draw a mark in the middle of their streamer (can be folded) and to write the value of this spot. Share the values students suggest and record these on the board to instigate discussion leading to the understanding that 50% or equivalent is appropriate. Pose the following question to students: The streamer was divided in the middle – what other ways could we describe this? Record students' responses to show diverse ways of referring to these portions.

3. Without measuring formally, ask students to make a mark the midpoint between 0 and 50% and 50% and 100% (can be folded) and record a value for each of these points. Discuss reasoning and language and recording for these points.

4. Ask students to construct a table reflecting their understanding of steps 1 to 3 of the activity. This could include the written word, common fraction, decimal fraction, and percentage notation together with a visual representation for each of these values (see below).

One half	$\frac{1}{2}$	$\frac{50}{100}$ or 0.50	50%
Three quarters	$\frac{3}{4}$	$\frac{75}{100}$ or 0.75	75%

A RECIPE FOR FRUIT SALAD

Specific teaching focus

To introduce simple proportion problems that introduce techniques for dealing with these situations through finding for 1 and multiplying or dividing.

Materials/resources required

- Create a chart displaying the recipe for fruit salad (see below)

A Recipe for Fruit Salad

For twelve people you need:

6 bananas.

12 oranges.

12 peaches.

24 strawberries.

600 ml of cream.

Chop fruit into bite sized pieces and place in a large bowl.

Serve with cream as desired.

How to implement

1. Open a discussion with students about catering and issues relating to adjusting recipes according to the number of people planned to be catered for. “What do you the cook need to think about? What strategies can be used?”

2. Show students the recipe for Fruit Salad. Pose the problem that they have 10 guests coming for dinner and dessert is to be fruit salad. How will they adjust the recipe? (Think adjust for 1 person then multiply by 10).

3. The fruit salad was such a success, that you decide that the class should make up the fruit salad recipe for a lunch time treat. If there are 25 students in the class what amounts of the ingredients should be purchased. (Think double that for ten and add half as much again)

4. Engage students in discussion relating to the strategies used to adjust the recipe down to 1 person through to increasing it to 25 people. Can students comment on how the quantities change and how they relate to each other, ie, for 1 person, the recipe is 1 twelfth, for 25 people, the recipe is double and half again of the quantities for 10 people.

GRANDMA SMITH'S FAVORITE CUPCAKE RECIPE

Learning goal

To develop techniques for dealing with proportion problems.

Listed below are the ingredients that are needed to make 12 of Grandma Smith's yummalicious cupcakes.

White flour	2 cups	*Butter or margarine*	$\frac{1}{4}$ cup
Vanilla essence	$\frac{1}{2}$ teaspoon	*Milk*	$\frac{3}{4}$ cup
Baking powder	$2\frac{1}{2}$ teaspoons	*Sugar*	$1\frac{1}{2}$ cups
Salt	1 teaspoon	*Chocolate*	50 grams

1. Calculate how much of each ingredient would be needed to make 24 cupcakes instead of 12 (make sure you show your understanding).

2. Calculate how much of each ingredient would be needed to make 6 cupcakes instead of 12 (make sure you show your understanding).

3. Jacqui, Grandma Smith's granddaughter, is a teacher. She wishes to make her grandmother's cupcakes for an excursion that her school is having. There are 42 students going on the excursion plus 6 adults (including Jacqui) and Jacqui would like to allow for 2 cupcakes per person (students and adults).

   a. Calculate how many cupcakes would need to be made.

   b. Calculate how much of each ingredient would be needed in order to make all of the cupcakes for the excursion (make sure you show your understanding).

TARGET PRACTICE

You need: 3 ten-sided dice (different colours are useful but not essential) and a copy of the game sheet for each student.

To play:

1. Take turns to throw the dice.
2. Choose 2 of the numbers thrown to make a number as close to the target number as possible.
3. Mentally calculate 'how close' and record in the space provided.
4. The winner is the player with the smallest total.

Numbers thrown	TARGET	Number Made	How close?
	74		
	3.5		
	1.9		
	5.7		
	3.5		
	4.7		
	9.3		
	0.8		

TOTAL