GROWING Mathematically

Multiplicative Thinking

Teaching Tasks (Zone 3)

Growing Mathematically is a project funded by the Australian Federal Government. It builds on materials that were developed as part of the RMIT Reframing Mathematical Futures II and the Scaffolding Numeracy in the Middle Years (SNMY) projects. SNMY content is currently hosted by the Victorian Department of Education.

Zone 3 Activities

The tasks listed on the initial page(s) are rich tasks from **reSolve** and **Maths300** that may be used with multi-zone groups. The tasks that follow these pages are suitable for students who are working in Zone 3.

ReSolve

Multiplication: Cartesian Product (Cartesian product)
Multiplication: Making Robots (Cartesian product)
Authentic Task: Bottle Flipping (Practical sharing situations)

Maths300

Cookie Count: Zones 2 – 5
This task introduces the idea of equal shares. Challenges occur at a range of levels with both whole numbers and fractions.

Halving Squares: Zones 2 – 3
This task begins as a spatial paper folding exercise involving halves to an open-ended exploration that leads to all sorts of mathematics.

Tackling Times Tables: Zones 2 – 6
This task builds the conceptual background using an array model and explores the number bonds associated with multiplication facts and the distributive law. It can be extended into algebraic reasoning using the structure of the array model.

Highest Number: Zones 1 - 3
This task starts out as a simple, well-known place value game with cards, but develops into many other learning outcomes, including statistics and probability.

Number Charts: Zones 1-4
This task covers additive strategies in Zones 1 and 2 and multiplicative strategies in Zones 3 and 4. The task seamlessly connects the inverse operations and uses reasoning strategies. Connects to Fractions and Fraction Charts and Algebra Charts.

Crazy Animals: Zones 3 – 8
This rich task introduces the notion of the Cartesian product, indices, algebraic representations, as well as linking to probability and statistics. Teachers can dip in and out of the many and varied activities within this task.

Fractions and Fraction Charts: Zones 3 – 5
This task looks at fractions as discrete groups and then at fraction charts where the inverse operations are used to reason through a problem. Other tasks in the maths300 suite such as Fraction Estimation and Rectangle Fractions investigate visualising and other representations of fractions.

Multiplication in a Table Format: Zones 3 – 6
The technique presented in this task provides a non-standard algorithm for multiplication that has a strong theoretical connection to arrays and the distributive property. It progresses naturally into expanding and factorising algebraic expressions.

Fraction Estimation: Zones 3 – 6
In this task students are challenged to improve their estimates of fractions by considering what constitutes the whole, how many equal parts it is to be divided into and how many of the parts need to be chosen. After students have done this with physical models there is software using various representations of fractions to use.

Rod Mats: Zones 3 – 4
This task explores equivalent fractions and operations involving fractions. The whole can be varied to represent different challenges.

Times 99: Zones 3 – 4
This task explores mental computation strategies for multiplying by 99 and then extends to include multiplying by numbers close to 99.

Make a Million: Zones 3 – 5
This task investigates how big a million is.

MULTIPLICATION STRATEGIES EXPLAINED*

Specific teaching focus

To introduce mental strategies for multiplication through doubling (2s facts), doubling and one more group (3s facts), double doubles (4s facts) and so on.

Materials/resources required

- Cuisenaire rods
- 10 sided dice

How to implement

*NOTE: these strategies are built on over time and would not be implemented in one lesson

1. Teacher rolls two 10 sided dice and reads aloud numbers rolled. E.g. “7, 2.”

2. Students choose either 7 twos or 2 sevens and discuss “which situation is easier to model and why.”

3. Students model both 7 twos and 2 sevens with Cuisenaire to explore the above issue and to demonstrate that these are the same.

4. Show through modelling with materials, and explore the following strategies:
   2s facts (2 ones, 2 twos, 2 threes … 2 of anything) are doubles.
   3s facts (3 ones, 3 twos, 3 threes … 3 of anything) are doubles and one more group.
   4s facts (4 ones, 4 twos, 4 threes … 4 of anything) are double, doubles.
   5s facts (5 ones, 5 twos, 5 threes … 5 of anything) are half of 10 of anything, or double doubles and one more group.
   6s facts (6 ones, 6 twos, 6 threes … 6 of anything) are double 3s.
   7s facts (7 ones, 7 twos, 7 threes … 7 of anything) are double 3s and one more group or 6s and one more group.
   8s facts (8 ones, 8 twos, 8 threes … 8 of anything) are double, double, doubles or double 4s.
   9s facts (9 ones, 9 twos, 9 threes … 9 of anything) are 10s less one group.

DOUBLE TROUBLE

Specific teaching focus

To introduce mental strategies for multiplication through doubling (2s facts), doubling and one more group (3s facts), double doubles (4s facts) and so on.

Materials/resources required

- Square grid paper
- 10 sided dice

How to implement

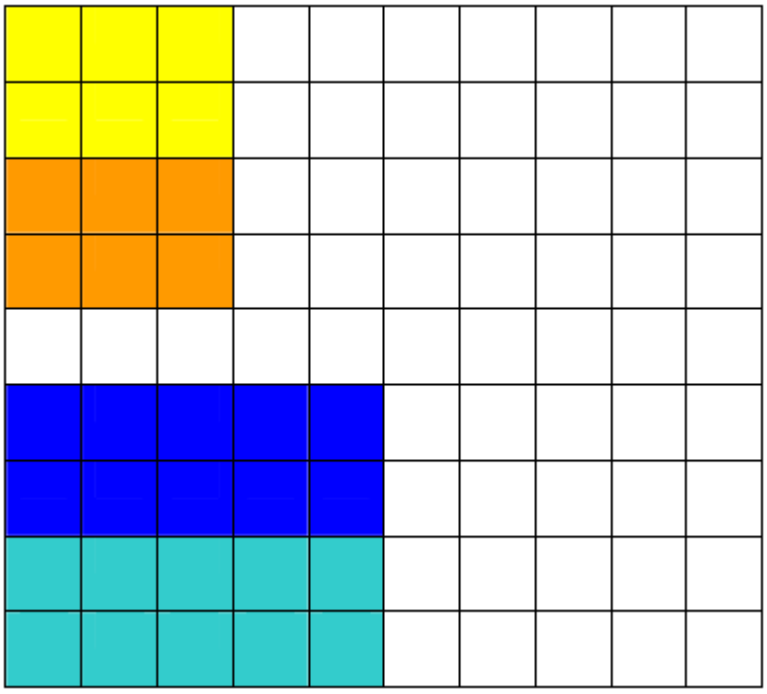
1. This activity is designed to visually demonstrate an efficient mental strategy based on doubling for learning 4s facts (4 ones, 4 twos, 4 threes … 4 of anything) THINK double 2s facts (ie. double doubles). 2s facts are best learnt through doubling. Understanding this concept demonstrates how the 4s facts strategy can be described as double doubles.

2. Teacher should first initiate a review of doubles to 10 using tens-frame (eg. 6 is double 3, 8 is double 4), then review doubles to 20 using two rows of a bead frame.

3. Using an enlarged version of 1cm grid paper, teacher demonstrates 2 threes by shading 2 rows of three as an array. Now shade 4 threes. Ask students, what do you notice? (4 threes are just double 2 threes).

4. Students can then shade their own arrays of various numbers chosen by teacher or by a throw of dice. Cuisenaire can also be used to show the double double strategy.

5. In pairs students take turns to throw a 10 sided dice, calculating 4 times the number thrown using the double double strategy (see below). Products can be shown on 1cm grid paper (winner is person with the largest amount of grid covered after say 6 throws) or alternatively products can be recorded and summed progressively (winner is person with the highest total).



MULTIPLYING MENTALLY

Specific teaching focus

Mental strategies for multiplication- doubling (2s facts), doubling and one more group (3s facts), double doubles (4s facts)

Materials/resources required

- MAB
- 10 sided dice

How to implement

1. Teacher rolls two 10 sided dice to create a two digit number. Students and teacher construct a model of this number with MAB. Ask students to think about double this number and model it with the MAB. Eg. a 2 and 3 are thrown, 23 is decided upon and doubled (see below).

THINK: double 23, 2 by 2 tens, 4 tens 40, 2 threes, 6, so 46.

2. Students work in pairs and throw their own dice to create 2 digit numbers. Do this a couple of times in order to visually demonstrate this more efficient mental strategy of dealing first with the tens, then with the ones.

3. Encourage students to identify and justify their thinking.

CARD MULTIPLICATION GAME

Specific teaching focus

To introduce mental strategies for multiplication through doubling (2s facts), doubling and one more group (3s facts), double doubles (4s facts) and so on.

Materials/resources required

- MAB
- Cuisenaire rods (label with appropriate numeral and word for the number to be represented eg, orange rod, with word ‘ten’ on two sides and the digit ‘10’ on two sides

- Playing cards (using cards 1-9 where Ace=1)
- Paper
- Pencils

How to implement

1. **Consolidate efficient strategies for multiplication facts through Cuisenaire:**
   Work in small groups with lots of Cuisenaire material available and ask students to select from the pile “two of anything” and describe. Eg. 2 sevens (shown with Cuisenaire) could be described as ‘double seven’. Ask students to select “two of something else“, and repeat discussion. Repeat selecting ‘three of something’ (double and one more group) and ‘four of something’ (double and double again) ensuring students model with Cuisenaire.

2. **Teacher models the procedure and thinking for the following situation:**
   Select two playing cards from the deck “Using these numbers, (Eg. 3 and 7) what two digit number can I make so that when I double it I get the largest number possible?” Discuss.
   Model the thinking for double 73: THINK double 7 tens, 14 tens, 140 and 6 more, 146. In pairs students repeat this activity together. To make this into a game, a point can be scored by the student with the highest number. Continue until one student has scored 5 points (or a set target).

Follow up suggestions

- The above activity can be implemented in the same way by creating the smallest number with the playing cards.

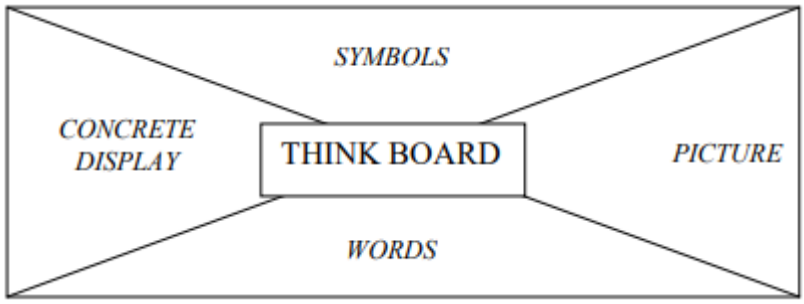
THINK BOARD

Specific teaching focus

To introduce place-value based strategies for informally solving problems involving single-digit by two-digit multiplication, mentally or in writing.

Materials/resources required

- MAB
- Cuisenaire rods
- Playing cards (using cards 1-9 where Ace=1)
- Large paper • Pencils
- Think board (teachers need to familiarise students with the four sections)



SYMBOLS: Algorithm showing the formal recording. Eg. 3x35.

PICTURE: Visual representation of the real life situation. Eg. A drawing of 3 buses with 35 children in them.

WORDS: Real life story describing the situation. Eg. “Students were transported to the school sports day in three buses, each bus carried 35 children.”

CONCRETE DISPLAY: Model using for example Cuisenaire or M.A.B.

How to implement

1. Draw a ‘Think Board’ on the board and discuss with students the four areas.

2. From the playing cards teacher selects 3 cards to be used to create a 2 digit by 1 digit multiplication situation. This will provide the focus for completing the Think Board modelled by the teacher. Work as a group to complete the four sections of the Think Board.

3. Students work in pairs and create their own multiplication problems using the playing cards. On the large sheet of paper they create a Think Board and fill in each section. Once each pair has completed all tasks, a whole group sharing session takes place.

Follow up suggestions

- Use the Think Board for division situations. Eg. 105 ÷ 3.

MULTIPLICATION & PLACE VALUE IDEAS

Specific teaching focus

To introduce place-value based strategies for informally solving problems involving single-digit by two-digit multiplication, mentally or in writing.

Materials/resources required

- Cuisenaire rods or MAB

How to implement

1. Using 6 x 14 and 14 x 6 as an example, teacher can show and explain (using Cuisenaire rods) the different ways in which equations can be modelled: Present as 6 fourteens and 14 sixes using Cuisenaire rods. Discuss which is more efficient/easier and why.

2. Invite students to share the different ways that this can be conceptualized. Eg. for 6 fourteens: 6 tens, 60 and 6 fours, 24 so total 84, or 3 fourteens, 42, double 42, 84, or 5 fourteens is half of 10 fourteens, 140, so 70 and one more fourteen, 84.

Follow up suggestions

- Explore efficient thinking for other situations. Eg. 8 x 23 is 8 twenties 160 and 8 threes 24, so 184, Or eight of anything is double, double, double so 46, 92, 184.

SCHOOL RUBBISH

Specific teaching focus

To introduce initial recording to support place-value for multiplication of single digit by 2 digit numbers.

Materials/resources required

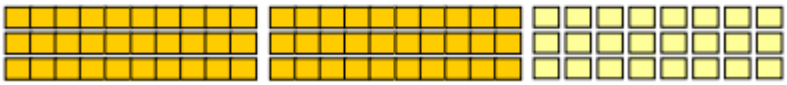
- Concrete materials representing ‘rubbish’
- MAB
- White board

How to implement

1. To introduce the use of formal recording when working out 2 digit by 1 digit multiplication, the teacher poses the following problem. “There are 28 students in a small rural school. If each person creates 3 pieces of rubbish from their lunch box, how many pieces of rubbish does the whole school produce in one day?”

2. Encourage students to use mental strategies to solve the problem first before moving on to formal recording. After this discussion, show students the formal method using the language to support formal recording as detailed below:

$$\begin{array}{r} 28 \\ \times\ 3 \\ \hline \end{array}$$



Use concrete materials such as MAB to model the region for 3 twenty eights.

Language to support formal recording:

- 3 ones by 8 ones, 24 ones, which is 2 tens and 4 ones, record 4 ones with the ones and 2 tens to regroup.
- 3 ones by 2 tens, 6 tens and 2 more tens, 8 tens, record with the tens.
- Ensure that you highlight the fact that the order of computation is reversed in formal notation. Eg. if you were doing this mentally, you’d encourage 3 x 2 tens, 6 tens (60) and 3 x 8, 24, so 84.

IF I KNOW

Specific teaching focus

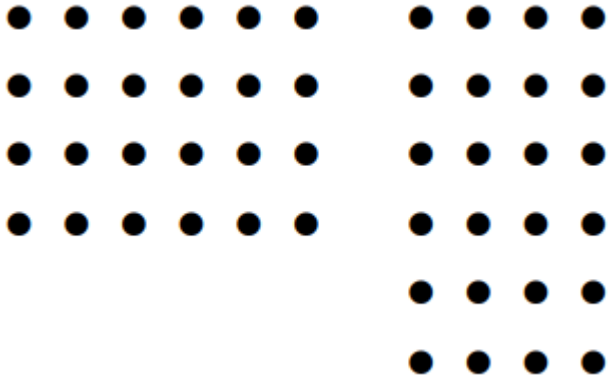
To introduce renaming number of groups by developing the ability to re-name composite numbers in terms of equal groups, 18 is 2 nines, 9 twos, 3 sixes, 6 threes.

Materials/resources required

- Counters

How to implement

1. Teacher and students discuss: “If I know 6 fours are 24, what else do I know?”
   Explore and record all possible options. Counters can be used to model these situations and students may note that a given model can be described in more than one way (see below).



Some possible responses are presented below:

4 sixes are 24	24 is 6 fours
24 divided by four is six	24 divided by six is four
4 threes and 4 threes	2 sixes and 2 sixes
6 and 6 and 6 and 6 is twenty four	4 + 4 + 4 + 4 + 4 + 4 = 24
4 x 6 = 24	24 = 6 x 4

   This list is by no means exhaustive and note that student responses can be recorded both formally and informally.

2. Students explore these ideas for other known facts. Working with a partner encourages discussion and reinforces associated language. (E.g. 48 is double 24, what facts can we deduce from our knowledge of 24. E.g. 4 twelves, 8 sixes, etc).

Follow up suggestions

- Build on the above to find more efficient strategies for two digit by one digit multiplication.
  E.g. teacher poses the following problem written on white/blackboard:
  - “If I know 6 x 4 = 24, how can this help me with 41 x 6?”
  - Lead discussion to 4 by 6 tens are 24 tens (240) and 6 more, 246.

IF I KNOW VARIATION

Goal

To identify different groupings of composite numbers in terms of equal groups (e.g. 18 is 2 nines, 9 twos, 3 sixes, 6 threes)

Equipment

- 'If I Know' cards
- 100 counters

Steps

1. Shuffle the 'If I Know' cards and place upside down in the middle of a table and have all group members sit around the table. Distribute the counters evenly.

2. Choose one person to begin the game by turning over a card in the middle and each game member represents a model of the number using counters.

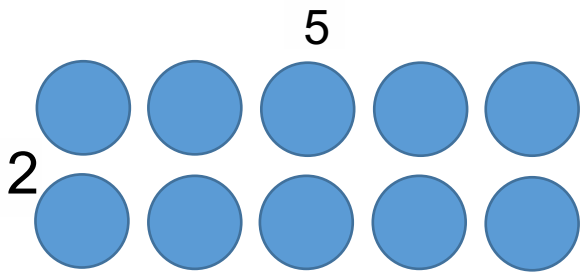
3. Players get 2 points if they make a model that nobody else makes, 1 point if they make a model that at least one other player makes, 0 points if they pass or if there is 3 or more with the same.

4. After everybody has shared, the person who turned the card over has a chance to gain extra points if they can show another model that nobody has shown (1 point for each model they can show).

5. Write down each of the possible groups into your book.

6. The next player has their go (Repeat steps 2 – 4).

7. Continue playing until someone reaches a designated winning score or until the cards are all played out.



2 fives (If 10 was turned over)

24

36

108

81

54

135

48

64

THE MAGICIAN'S COSTUMES

Specific teaching focus

To introduce cartesian product idea to systematically list or determine all possible options.

Materials/resources required

- Large sheets of paper for recording
- Costumes similar to ones identified in the problem (if desired)

How to implement

1. Introduce Sam the Stupendous, the magician. For the magician's next show, there will be a number of costume changes. Sam always wears a hat, a cape and a magic wand and can choose from the following:

Cape	Hat	Magic wand
Black velvet	Top hat	Gold
Red satin	Beret	Silver

2. Pose the question: "How many different costumes can Sam wear?"

3. Students work in small groups to respond to the above question. They record their thinking on a large piece of paper (using words and/or pictures). All efficient strategies are accepted and encouraged. E.g. lists, tables, diagrams.

   As a class, share the group work and explain their solutions. Eg. "I know I have all possible options because …"